Spin Qubits and Scalable 2D Architectures

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Outline

- spin qubits and quantum dots: GaAs and others

- long-distance spin-spin coupling (floating gates)

  ➔ **scalable 2D architecture**

- (Exotic) Bound states in CDW-wires as quantum dots

Quantum Computing (basics)

• basic unit: **qubit** → any state of a quantum two-level system

$$|\Psi\rangle = a|1\rangle + b|0\rangle$$

"natural" candidate: **electron spin**

• quantum computation:

1) prepare N qubits (input)
2) apply unitary transformation in 2^N-dim. Hilbert space
→ computation
3) measure result (output)

• quantum computation faster than classical:

- factoring algorithm (Shor 1994): exp N → N^2
- database search (Grover 1996): N → $N^{1/2}$
- quantum simulations

...

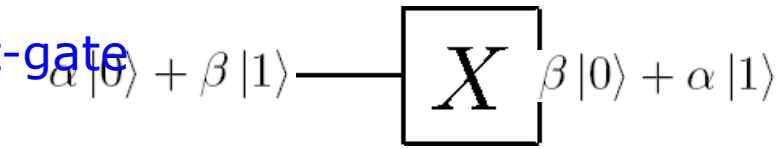
Universal Set of Quantum Gates

A. Barenco et al., Phys. Rev. A 52, 3457 (1995)

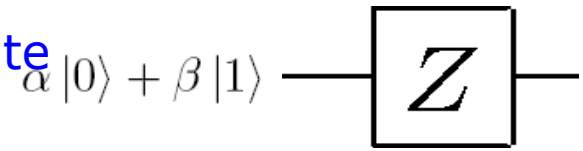
Single-qubit operations and a two-qubit gate that generates entanglement are sufficient for **universal quantum computation**:

Single-qubit gates

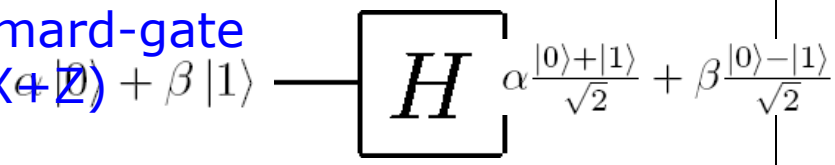
Not-gate



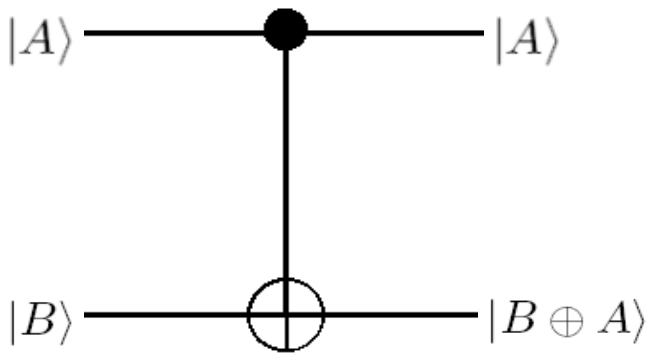
Z-gate



Hadamard-gate
(H= X+Z)



CNOT gate



⇔ entanglement

Electron qubit: spin better than charge

due to longer relaxation/decoherence* times

$$\tau_{\phi}^{spin} >> \tau_{\phi}^{charge}$$

10ns -1s

1 ns

GaAs

Awschalom et al., ’97
Tarucha et al., ‘02
Kouwenhoven/Vandersypen et al., ’03-’09
Abstreiter et al., ’04/08; Warburton et al., ‘09
Zumbuhl et al., '08
Marcus, Yacoby et al.,’05- ’11 (T_2 ~ 270 μs)

‘mesoscopics’
Fujisawa et al. ‘03
Marcus et al. ‘01

➔ natural choice for qubit: spin ½ of electron

*) theory: T_2 ~ T_1 for single spin in GaAs dot (‘everything optimized’)

PHYSICAL REVIEW A VOLUME 57, NUMBER 1 JANUARY 1998

Quantum computation with quantum dots

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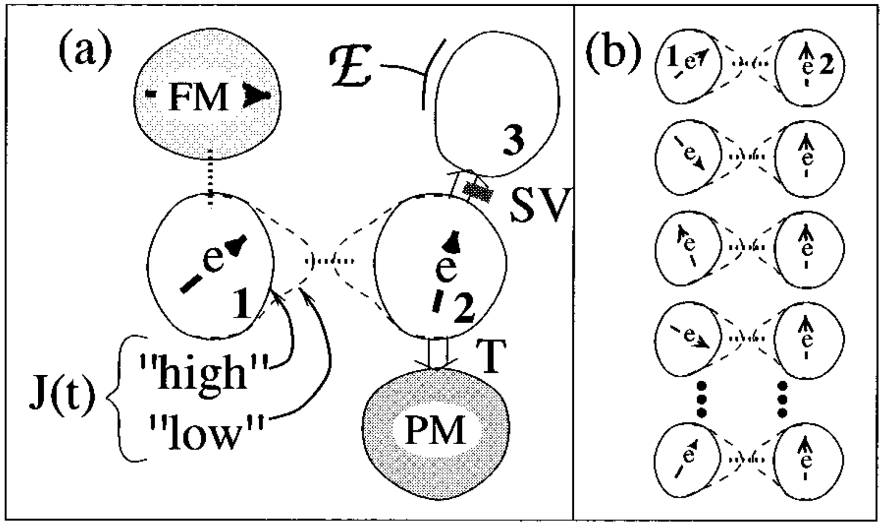
(Received 9 January 1997; revised manuscript received 22 July 1997)



'all-electrical scheme'

$$U_{XOR} = e^{i\frac{\pi}{2}S_1^z}\, e^{-i\frac{\pi}{2}S_2^z}\, U_{SW}^{1/2}\, e^{i\pi S_1^z}\, U_{SW}^{1/2}$$

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Times Cited: > 3050

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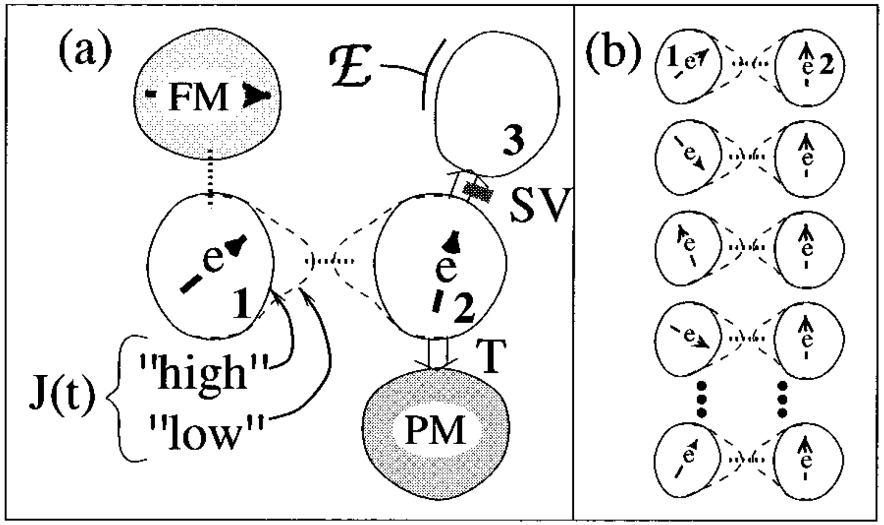

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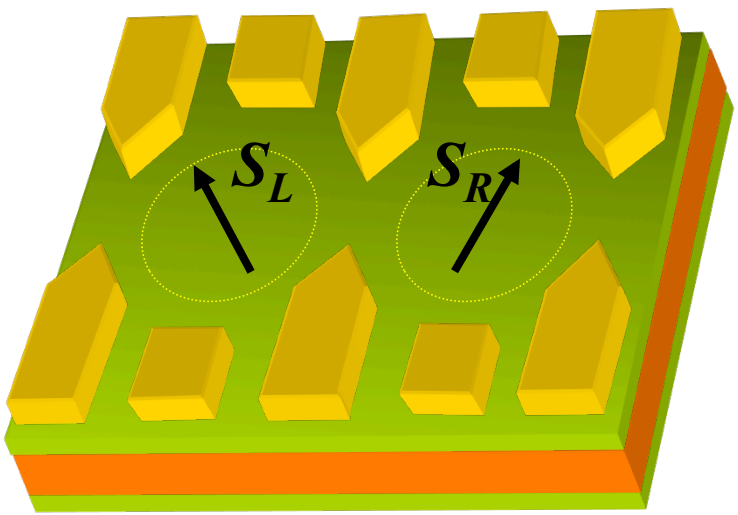
Electric fields vs. Magnetic fields

- Strong electric fields easy to produce (gates, STM-tips, etc)
- Fast switching of electric fields (picoseconds)
- Easy to apply electric fields locally and on nanoscale

- Strong magnetic (ac) fields hard to produce
- Slow switching of magnetic fields (nanoseconds)
- Hard to apply magnetic fields locally and on nanoscale

Quantum Processor for Spin-Qubits

DL & DiVincenzo, PRA **57** (1998)



Spin 1/2 of electron = qubit

Quantum Processor for Spin-Qubits

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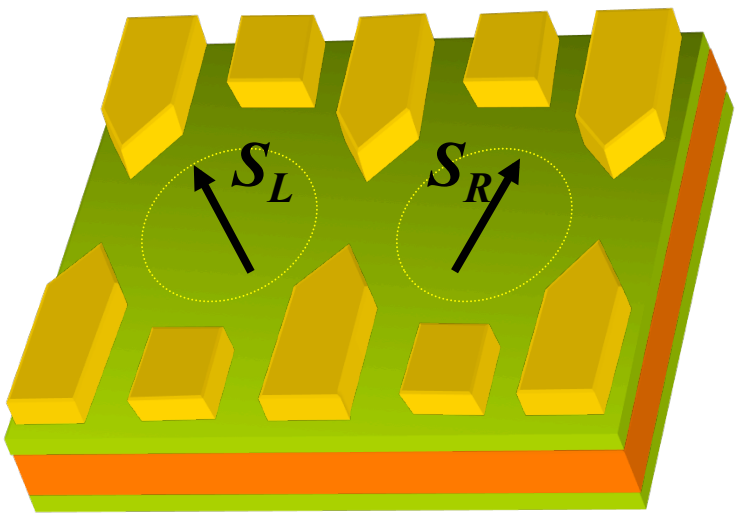
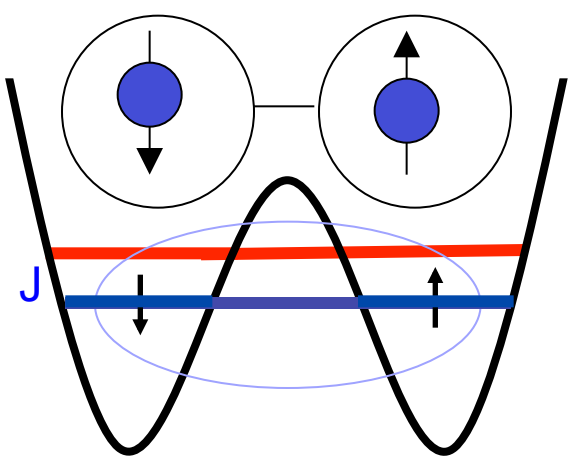


2 quantum dots, each with
1 electron-spin (= qubit)

Key idea:
all-electrical control of spins
➔ potentially scalable



artificial hydrogen molecule ➔ exchange splitting $J \sim t^2/U$

➔ 'CNOT quantum gate'

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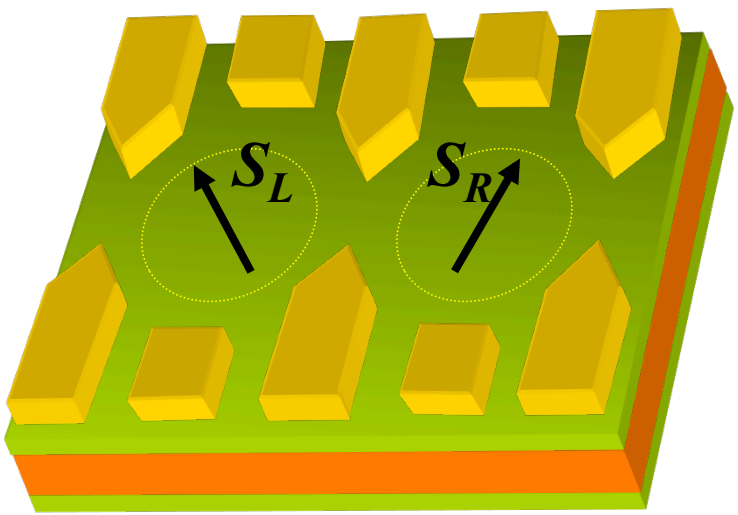
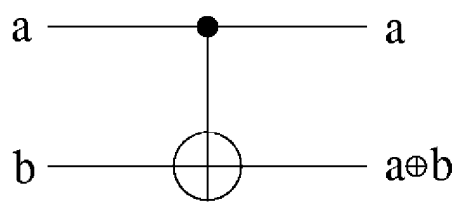


$$H(t) = J(t)\, \boldsymbol{S}_L \cdot \boldsymbol{S}_R$$

➔ CNOT (XOR) gate

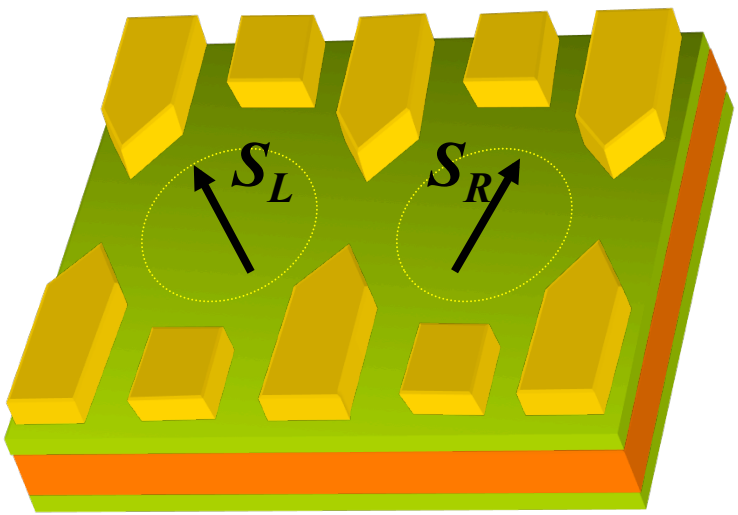


$$\Leftrightarrow \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \sigma_x \end{pmatrix} \begin{array}{c} \text{a b} \\ \left\{\begin{matrix} \uparrow\uparrow \\ \uparrow\downarrow \\ \downarrow\uparrow \\ \downarrow\downarrow \end{matrix}\right\} \end{array} = \left\{\begin{matrix} \uparrow\uparrow \\ \uparrow\downarrow \\ \downarrow\downarrow \\ \downarrow\uparrow \end{matrix}\right\}$$

$$U(\tau_s) = T e^{-i\int_0^{\tau_s} H'(t)dt}, \quad J \neq 0 \;\; during \;\; \tau_s$$

Quantum Processor for Spin-Qubits

DL & DiVincenzo, PRA **57** (1998)



$$H(t) = J(t)\, \boldsymbol{S_L} \cdot \boldsymbol{S_R}$$

➔ CNOT (XOR) gate

$$U_{XOR} = e^{i\frac{\pi}{2}S_1^z}\, e^{-i\frac{\pi}{2}S_2^z}\, U_{SW}^{1/2}\, e^{i\pi S_1^z}\, U_{SW}^{1/2}$$

$$U_{SW} : \uparrow\downarrow \;\Rightarrow\; \downarrow\uparrow$$

sqrt-of-swap: $U_{SW}^{1/2} : \uparrow\downarrow \;\Rightarrow\; \uparrow\downarrow + e^{i\alpha}\downarrow\uparrow$

switching time: 180 ps

Petta, Marcus, Yacoby *et al.*, Science, 2005

Serial vs. Parallel gate

I. Serial gate: LD, PRA 57, 120 (1998)

$$H(t) = J(t)\mathbf{S}_1 \cdot \mathbf{S}_2 \quad \text{and} \quad H_Z(t) = \mathbf{b}_1(t) \cdot \mathbf{S}_1 + \mathbf{b}_1(t) \cdot \mathbf{S}_2$$

$$U_{SW}^{1/2} = \exp(i\int_0^{\tau_s} dt J(t)\mathbf{S}_1 \cdot \mathbf{S}_2) \ , \quad if \quad \int_0^{\tau_s} dt J(t) = \pi/2 + 2\pi n$$

$$U_{XOR} = e^{-i(\pi/2)S_2^y}[e^{i(\pi/2)S_1^z} e^{-i(\pi/2)S_2^z} U_{SW}^{1/2} e^{+i\pi S_1^z} U_{SW}^{1/2}]e^{i(\pi/2)S_2^y}$$

➔ need 7 pulses (5 for CPF)

Serial vs. Parallel gate

II. Parallel gate: Burkard *et al.*, PRB 60, 11404 (1999)

$$H(t) = J(t)\mathbf{S}_1 \cdot \mathbf{S}_2 + \mathbf{b}_1(t) \cdot \mathbf{S}_1 + \mathbf{b}_2(t) \cdot \mathbf{S}_2$$

$$U_{CPF} = \exp(i\int_0^{\tau_s} dt H(t))$$

only 1 pulse for CPF !

$$if \quad \int J = \pi/2, \ and \ \int b_{1/2}^z = \pi(1 \pm \sqrt{3})/4$$

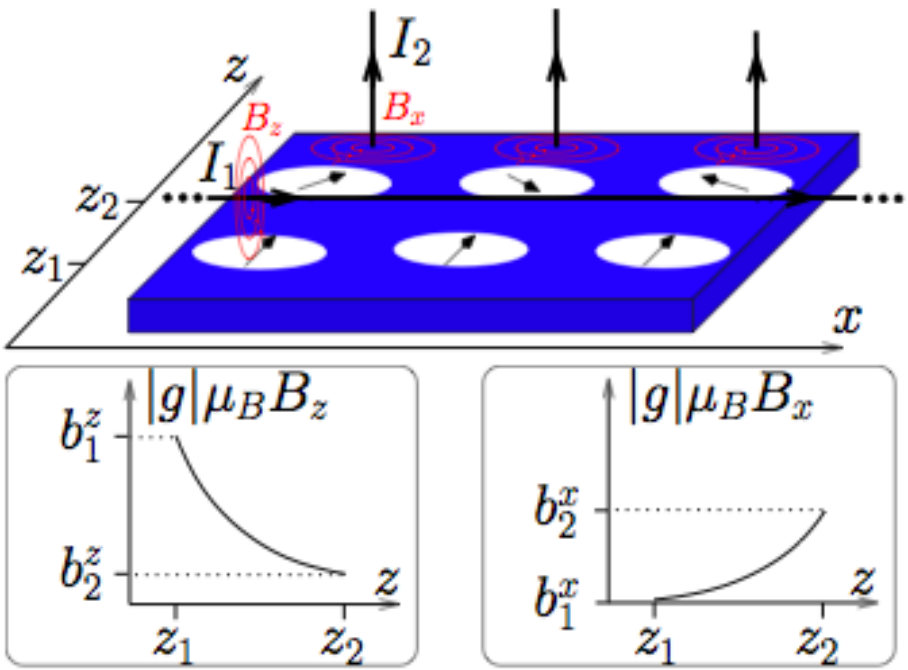
$$U_{XOR} = e^{-i(\pi/2)S_2^y} U_{CPF} e^{i(\pi/2)S_2^y}$$

➔ need only 3 pulses

Implementation scheme: Meunier *et al.*, PRB 83, 121403 (2011)

Single-Spin Rotations by Exchange only

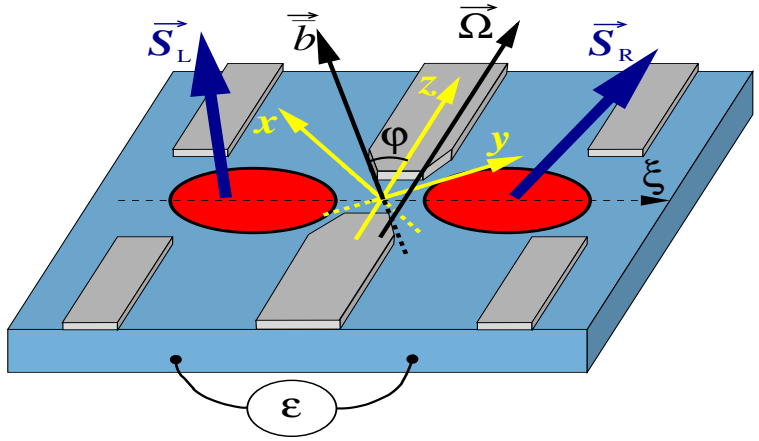
Coish & DL, Phys. Rev. B **75**, 161302 (2007)



Requires auxiliary spins,
Zeeman gradient & exchange
➔ fast switching times (1ns)
with high fidelity (< 10^{-3})

Two-Electron Double Quantum Dot

Stepanenko, Rudner, Halperin, DL, arXiv:1112.1644



Interactions that conserve total spin $\boldsymbol{S} = \boldsymbol{S}_L + \boldsymbol{S}_R$:

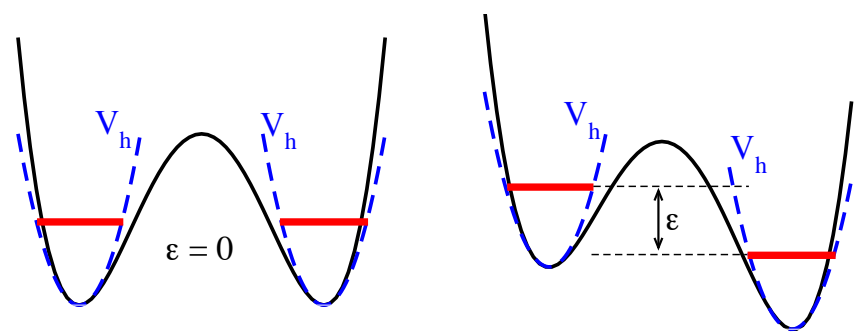
Burkard, Loss, DiVincenzo PRB 1999

$$H_0 = U \sum_{j=R,L} n_j (n_j - 1) + \frac{n_L n_R}{2}[(2 - \mathbf{S}^2)V_+ + \mathbf{S}^2 V_-]$$
$$+ X \sum_{\sigma=\uparrow,\downarrow} (c^+_{L\sigma} c^+_{L\bar{\sigma}} c_{R\sigma} c_{R\bar{\sigma}} + \text{h.c.})$$
$$+ t \sum_{i \neq j,\, \sigma} (c^+_{i\sigma} c_{j\sigma} + \text{h.c.}) + \frac{\varepsilon}{2}(n_L - n_R) \quad .$$

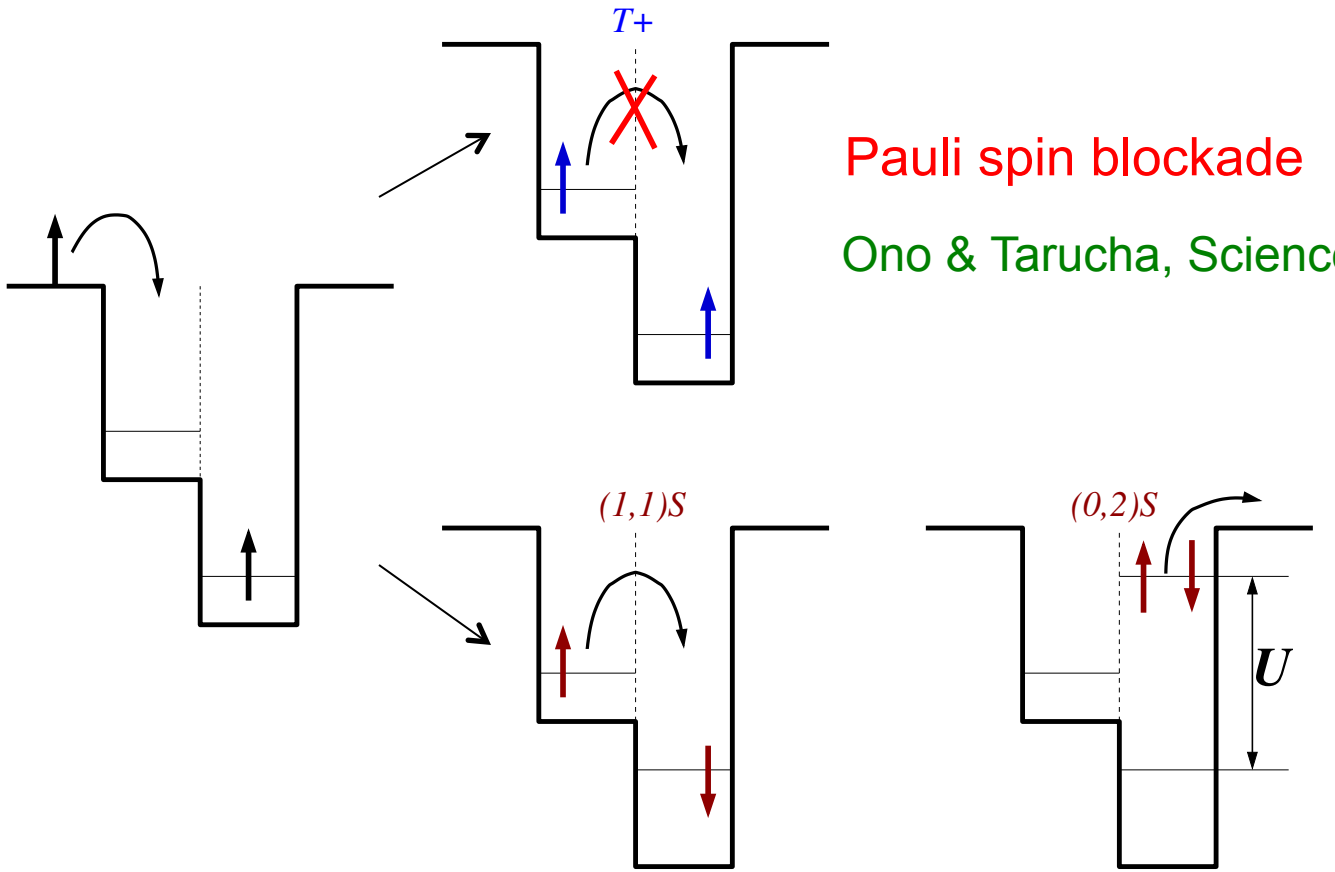
- on-site repulsion U
- direct exchange $V_\pm$
- correlated hopping X (very weak)
- spin-independent hopping t

- detuning ε
- Zeeman coupling $\boldsymbol{b}$

} control parameters



Spin-conserving interactions – ‘spin via charge’



Pauli spin blockade

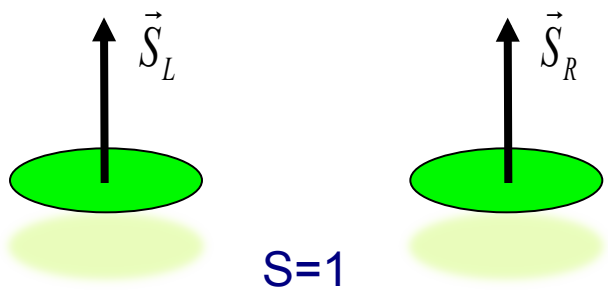
Ono & Tarucha, Science (2002)

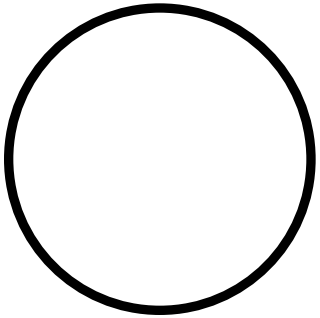
Spin blockade: the charge configuration depends on the spin state

spin-conserving interactions only → spin blockade *cannot* be lifted

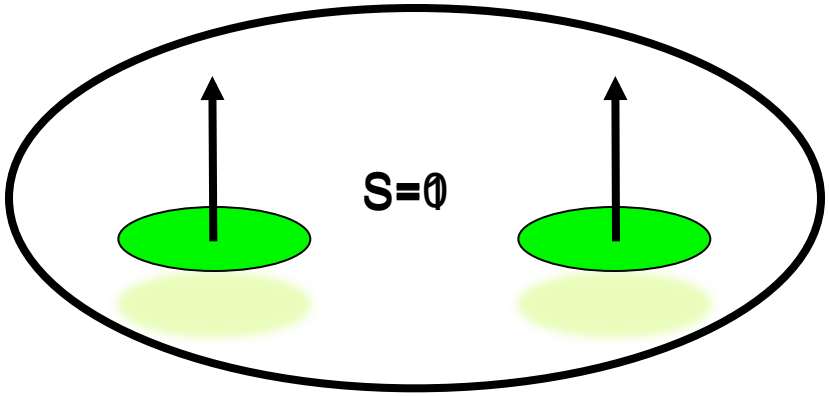
Spin non-conserving interactions

Spin non-conserving interactions couple triplets to singlets of different charge configurations



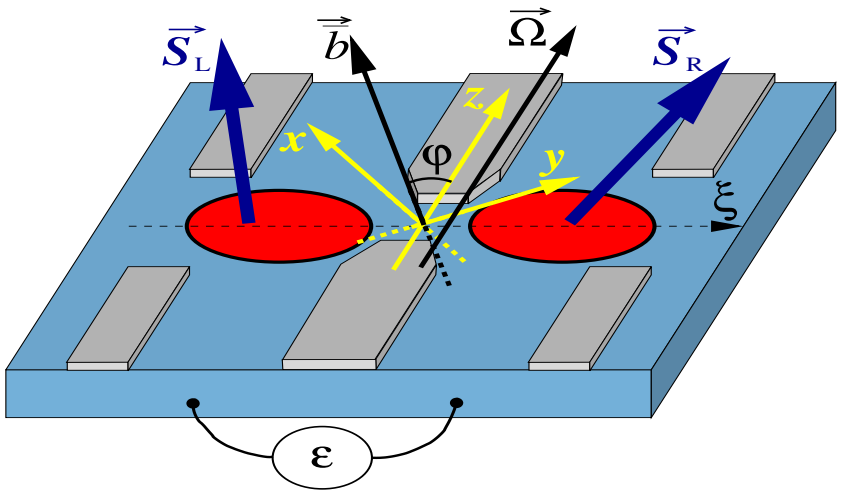


2. Nuclear hyperfine interaction



By controling charge state of the anti-crossing, we can make one or the other interaction dominant

I. Spin-orbit interaction (SOI)



Structure inversion asymmetry – Rashba SOI:

Bulk inversion asymmtery – Dresselhaus SOI:

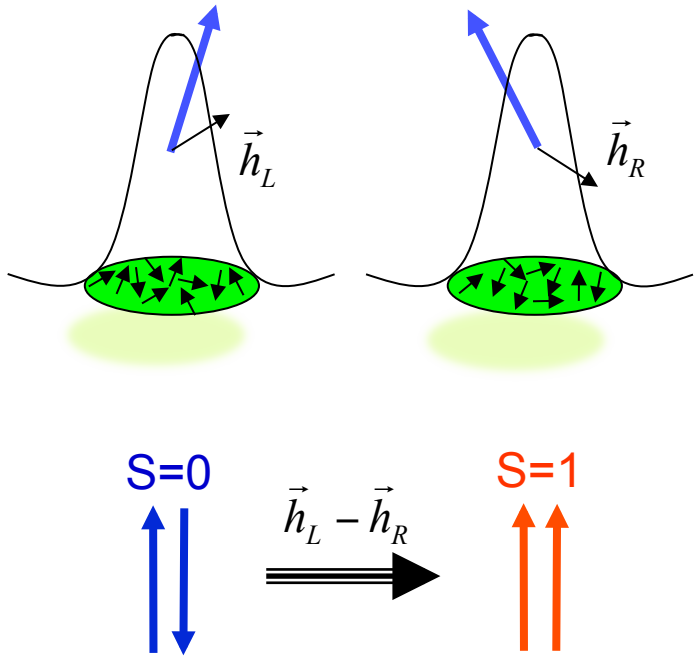
$$H_{\rm SO} = \alpha\left(p_{x'}\sigma_{y'} - p_{y'}\sigma_{x'}\right) + \beta\left(-p_{x'}\sigma_{x'} + p_{y'}\sigma_{y'}\right)$$ (001) plane

For two electrons in the lowest dot states:

$$H_{\rm SO} = \frac{i}{2}\boldsymbol{\Omega}\cdot\sum_{\alpha,\beta=\uparrow\downarrow}\left(c^\dagger_{{\rm L}\alpha}\boldsymbol{\sigma}^{\alpha\beta}c_{{\rm R}\beta} - {\rm h.c.}\right)$$

SOI vector $\boldsymbol{\Omega}$: depends on geometry and dot confinement

II. Nuclear hyperfine interaction



Hyperfine interaction of electron spin with nuclear spins on same dot:

$$H_{\rm nuc} = \sum_{\alpha={\rm LR}} \mathbf{h}_\alpha \cdot \mathbf{S}_\alpha$$

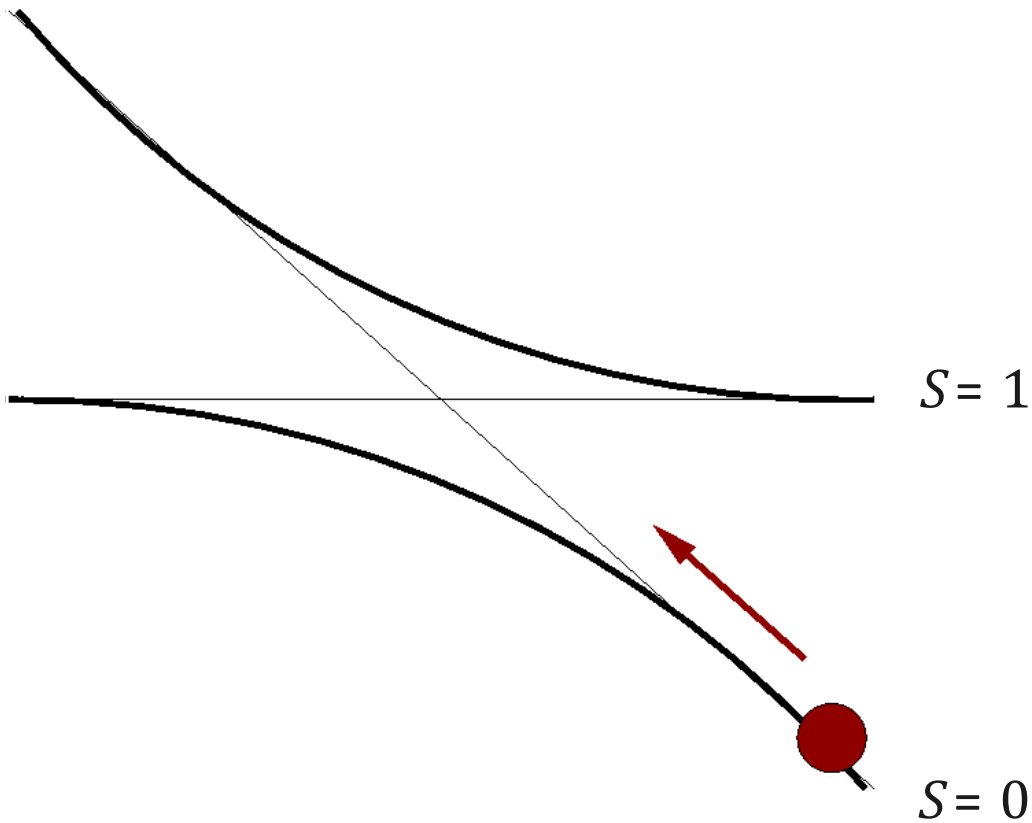
$$\mathbf{h}_{\rm L} = A \sum_{\mathbf{r}\in L} |\psi(\mathbf{r})|^2 \mathbf{I}_{\mathbf{r}}$$

Hyperfine interaction acts as an inhomogeneous magnetic field (it does not affect orbital states of electrons):

$$\boxed{H_{\rm nuc} + H_{\rm Z} = -\bar{\mathbf{b}} \cdot (\mathbf{S}_{\rm L} + \mathbf{S}_{\rm R}) - \delta\mathbf{b} \cdot (\mathbf{S}_{\rm L} - \mathbf{S}_{\rm R})\,.}$$

Spin-control via spin blockade

Petta et al., Science 2005; Nowack et al., Science 2007; Reilly et al., Science 2008; Bluhm et al., Nat. Phys. 2011; Brunner et al., PRL 2011; ...



Detune the dots strongly to prepare a (2,0)S singlet

Spin-control via spin blockade

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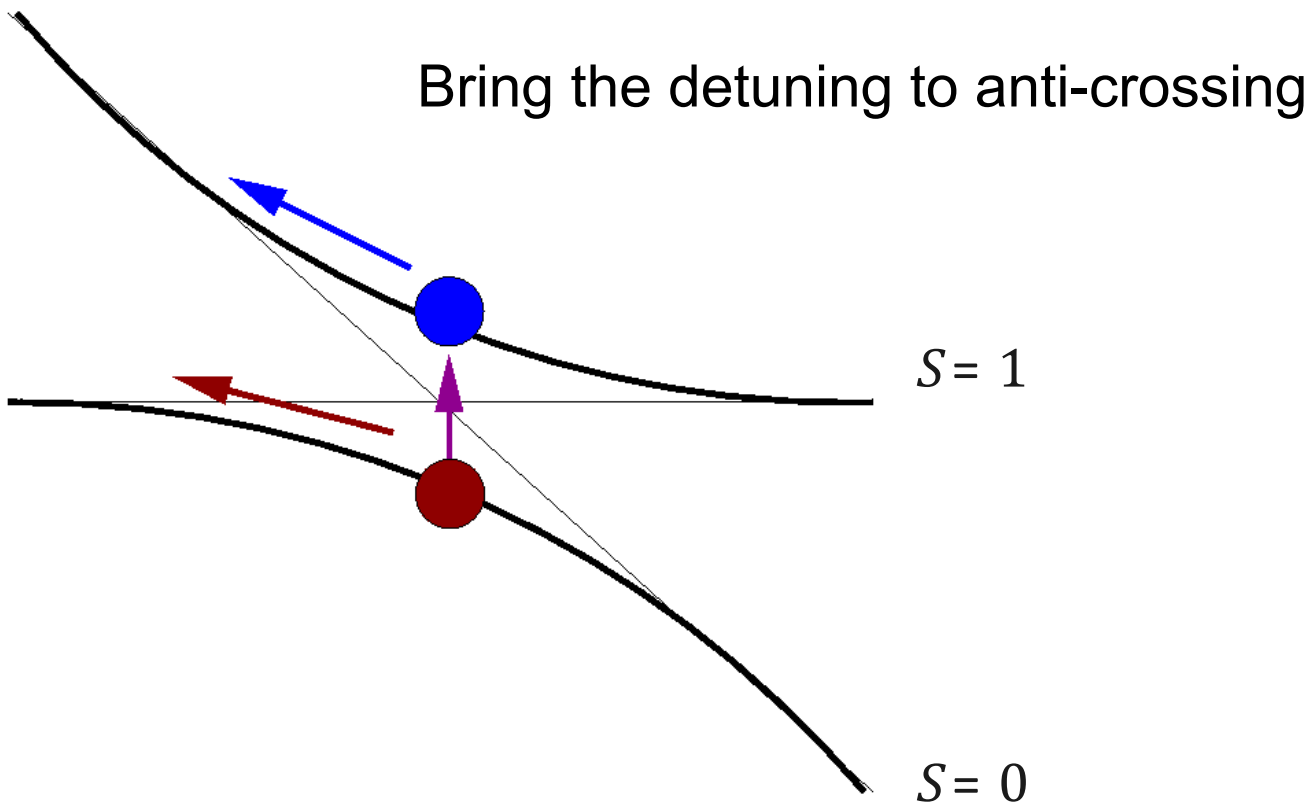

Subsequent evolution depends on whether there was a spin-flipping transition at the crossing or not

Spin-control via spin blockade

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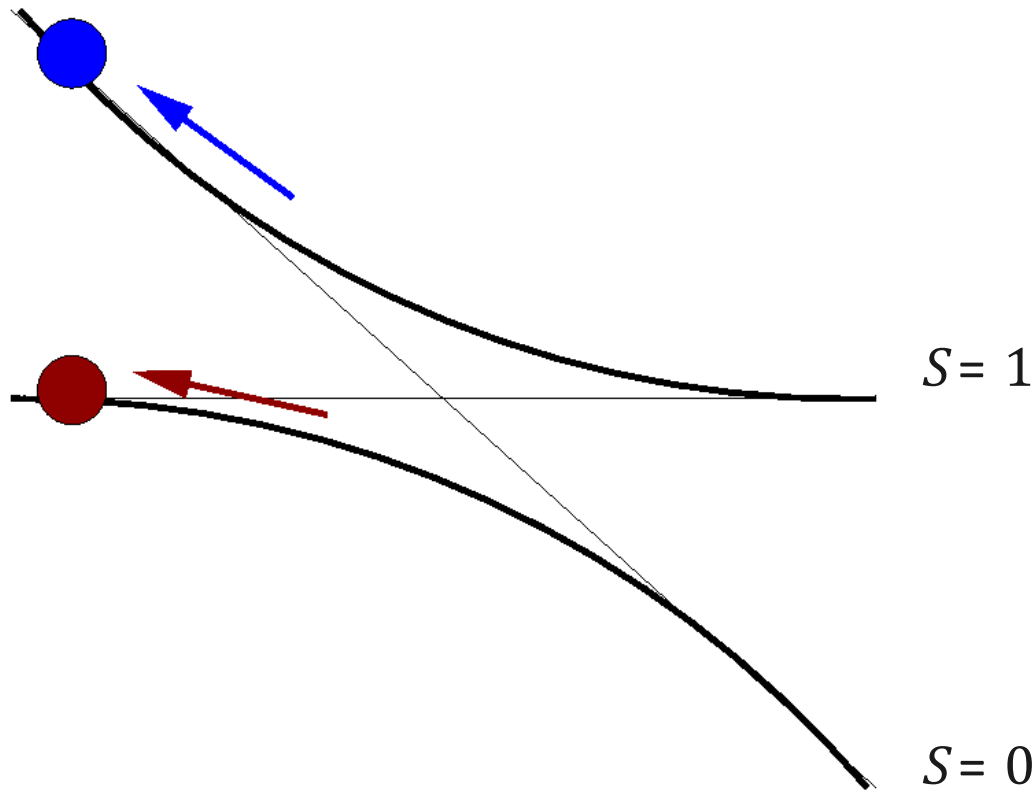
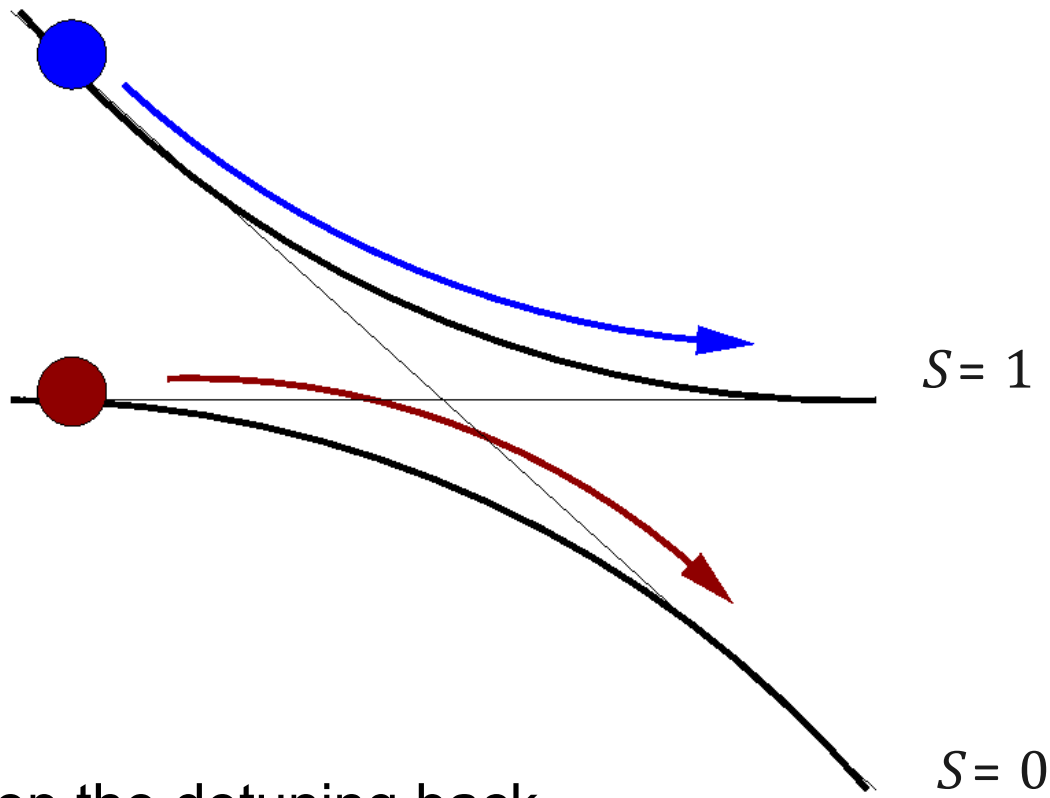


After anticrossing, the state still depends on the possible spin flip

Spin-control via spin blockade

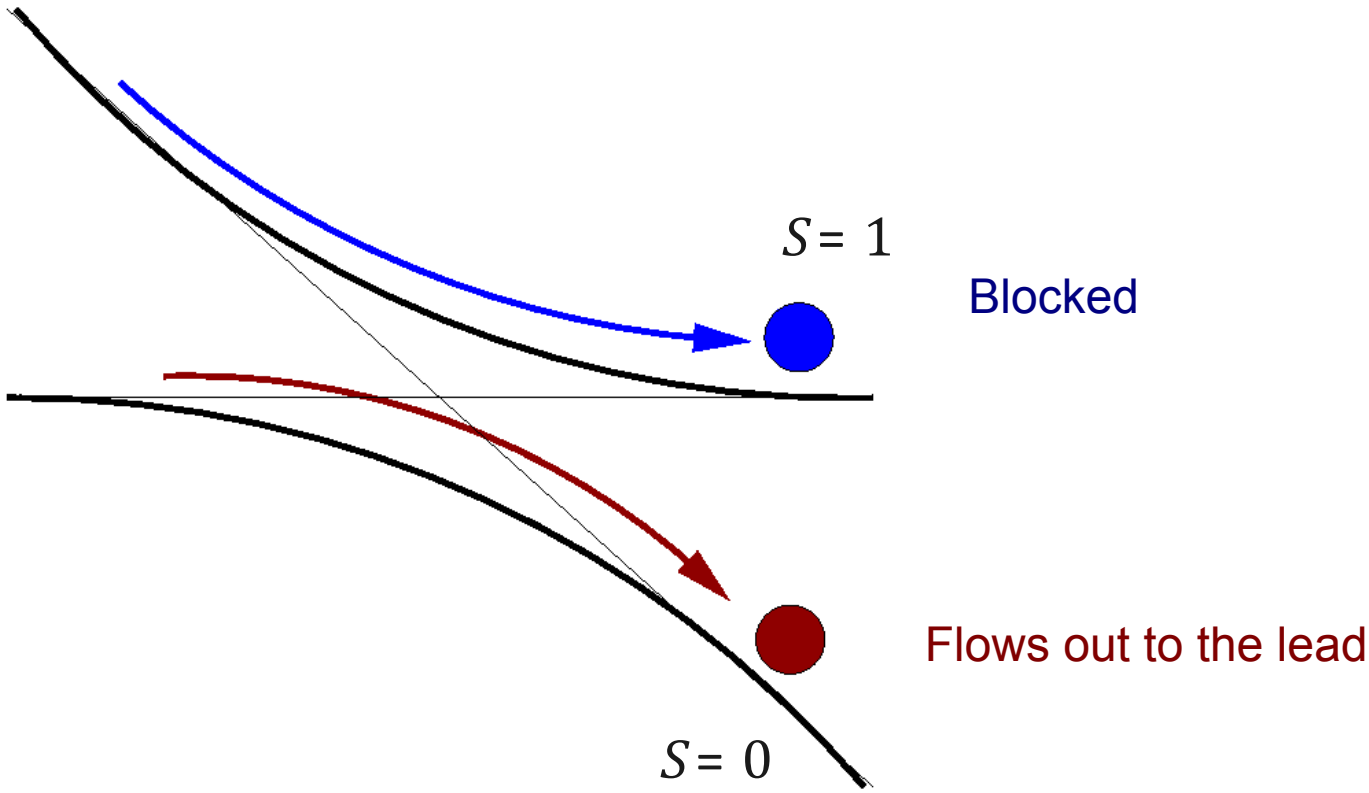
Petta et al., Science 2005; Nowack et al., Science 2007; Reilly et al., Science 2008; Bluhm et al., Nat. Phys. 2011; Brunner et al., PRL 2011; ...



Quickly sweep the detuning back

Spin-control via spin blockade

Petta et al., Science 2005; Nowack et al., Science 2007; Reilly et al., Science 2008; Bluhm et al., Nat. Phys. 2011; Brunner et al., PRL 2011; ...



Transferred charge depends on the change of spin at anticrossing

Double Dot Hamiltonian (6x6)

Stepanenko, Rudner, Halperin, DL, arXiv:1112.1644

keep 6 lowest states: 3 singlets and 3 triplets

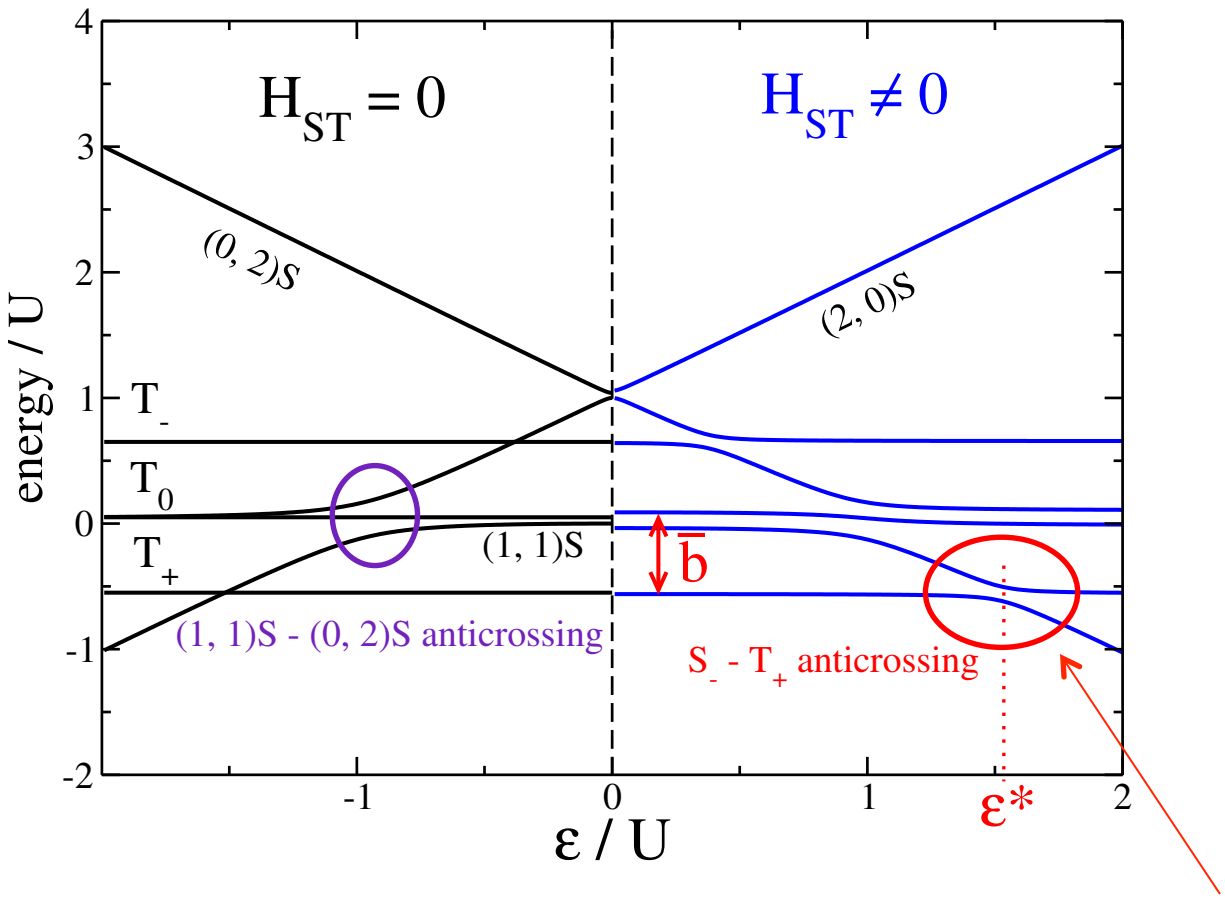
$$H = \begin{pmatrix} U-\varepsilon & X & -\sqrt{2}t & 0 & -i\sqrt{2}\Omega & 0 \\ X & U+\varepsilon & -\sqrt{2}t & 0 & -i\sqrt{2}\Omega & 0 \\ -\sqrt{2}t & -\sqrt{2}t & V_+ & -\sqrt{2}\left(\delta b_x - i\delta b_y\right) & 2\delta b_z & \sqrt{2}\left(\delta b_x + i\delta b_y\right) \\ 0 & 0 & -\sqrt{2}\left(\delta b_x + i\delta b_y\right) & V_- + 2\bar{b}_z & \bar{b}_x\sqrt{2} & 0 \\ i\sqrt{2}\Omega & i\sqrt{2}\Omega & 2\delta b_z & \bar{b}_x\sqrt{2} & V_- & \bar{b}_x\sqrt{2} \\ 0 & 0 & \sqrt{2}\left(\delta b_x - i\delta b_y\right) & 0 & \bar{b}_x\sqrt{2} & V_- - 2\bar{b}_z \end{pmatrix}$$

I. Diagonalize numerically: get exact spectrum as function of detuning ε

II. Solve analytically around anti-crossings: get full parameter dependences

Double Dot Spectrum (exact)

Stepanenko, Rudner, Halperin, DL, arXiv:1112.1644



Larger b-field increases (2,0)S component at ST- anticrossing

Singlet-Triplet mixing Hamiltonian

$$H = \begin{pmatrix} H_{\mathrm{SS}} & H_{\mathrm{ST}} \\ H_{\mathrm{TS}} & H_{\mathrm{TT}} \end{pmatrix}$$

$$H_{\mathrm{TT}} = \mathrm{diag}\left(V_- - |\bar{\mathbf{b}}|, V_-, V_- + |\bar{\mathbf{b}}|\right)$$

$$H_{\mathrm{ST}} = H_{\mathrm{ST}}^{\mathrm{SO}} + H_{\mathrm{ST}}^{\delta \mathrm{b}}$$

Spin orbit interaction:

$$H_{\mathrm{ST}}^{\mathrm{SO}} = i\Omega \begin{pmatrix} -\sin\varphi & -\sqrt{2}\cos\varphi & \sin\varphi \\ \cos\psi\sin\varphi & \sqrt{2}\cos\psi\cos\varphi & -\cos\psi\sin\varphi \\ -\sin\psi\sin\varphi & -\sqrt{2}\sin\psi\cos\varphi & \sin\psi\sin\varphi \end{pmatrix}$$

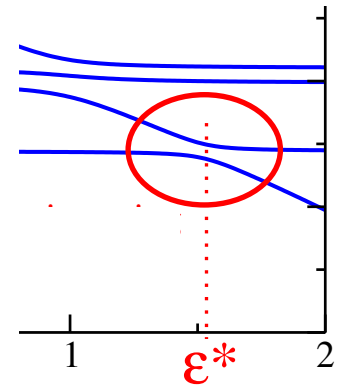
Hyperfine interaction:

$$H_{\mathrm{ST}}^{\delta\mathbf{b}} = \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{2}\left(i\delta b_y + \delta\mathbf{b}\cdot\mathbf{e}'\right)\sin\psi & 2\left(\delta\mathbf{b}\cdot\mathbf{e}\right)\sin\psi & \sqrt{2}\left(i\delta b_y - \delta\mathbf{b}\cdot\mathbf{e}'\right)\sin\psi \\ \sqrt{2}\left(i\delta b_y + \delta\mathbf{b}\cdot\mathbf{e}'\right)\cos\psi & 2\left(\delta\mathbf{b}\cdot\mathbf{e}\right)\cos\psi & \sqrt{2}\left(i\delta b_y - \delta\mathbf{b}\cdot\mathbf{e}'\right)\cos\psi \end{pmatrix}$$

Splitting at singlet-triplet crossing

Stepanenko, Rudner, Halperin, DL, arXiv:1112.1644

$$\Delta^*_{\mathrm{ST}} \equiv \Delta_{\mathrm{ST}}(\varepsilon^*(\bar{b}), \bar{b}, \varphi) = \min_{\varepsilon} \Delta_{\mathrm{ST}}(\varepsilon, \bar{b}, \varphi)$$



minimum singlet-triplet splitting as the detuning ε is swept in a magnetic field b

Detuning ε controls the orbital state of the crossing singlet

$$|S_-\rangle = \cos\psi|(1,1)S\rangle + \sin\psi|(0,2)S\rangle$$

where ψ is mixing angle of (0,2)S and (1,1)S singlets:

$$\cos 2\psi = \frac{U - V_+ - \varepsilon}{\sqrt{(U - V_+ - \varepsilon)^2 + 8t^2}} \qquad \sin 2\psi = \frac{2\sqrt{2}t}{\sqrt{(U - V_+ - \varepsilon)^2 + 8t^2}}$$

Effective Hamiltonian at singlet-triplet anti-crossing

Stepanenko, Rudner, Halperin, DL, arXiv:1112.1644

$$\boxed{v_{\mathrm{ST}} = -ip \sin\frac{\Psi}{2}\sin\phi + \frac{1}{2}\left(\delta b_z - \delta b_+\right)\cos\frac{\Psi}{2};}$$

angle between the magnetic field ***B*** and in-plane normal to SOI-field **Ω**

ψ: mixing angle of (0,2)S and (1,1)S singlets:

$$\cos 2\psi = \frac{U - V_+ - \varepsilon}{\sqrt{\left(U - V_+ - \varepsilon\right)^2 + 8t^2}} \qquad \sin 2\psi = \frac{2\sqrt{2}t}{\sqrt{\left(U - V_+ - \varepsilon\right)^2 + 8t^2}}$$

Consequence:

Average spin-momentum transfer between electrons and nuclei is quantized:

Rudner and Levitov, PRB (2010)

$$|v_{\mathrm{SO}}| > |v_{\mathrm{nuc}}| \longrightarrow \langle \Delta m \rangle = 0$$

$$|v_{\mathrm{SO}}| < |v_{\mathrm{nuc}}| \longrightarrow \langle \Delta m \rangle = 1$$

Splitting at Anti-Crossing

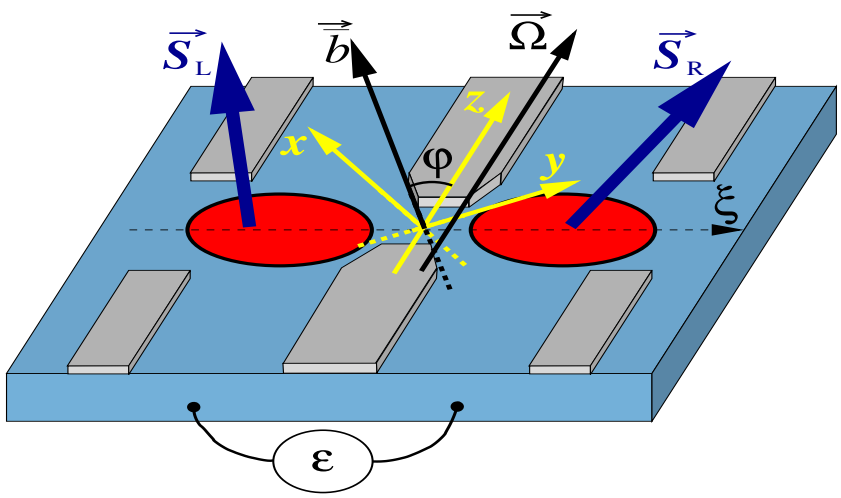
Stepanenko, Rudner, Halperin, DL, arXiv:1112.1644

$$\Delta^*_{\mathrm{ST}} = 2\left|-i\Omega\sin\varphi\sin\psi + \sqrt{2}\left(\delta\mathbf{b}\cdot\mathbf{e}' + i\delta b_y\right)\cos\psi\right|$$

angle φ between **B**-field and spin-orbit vector **Ω**

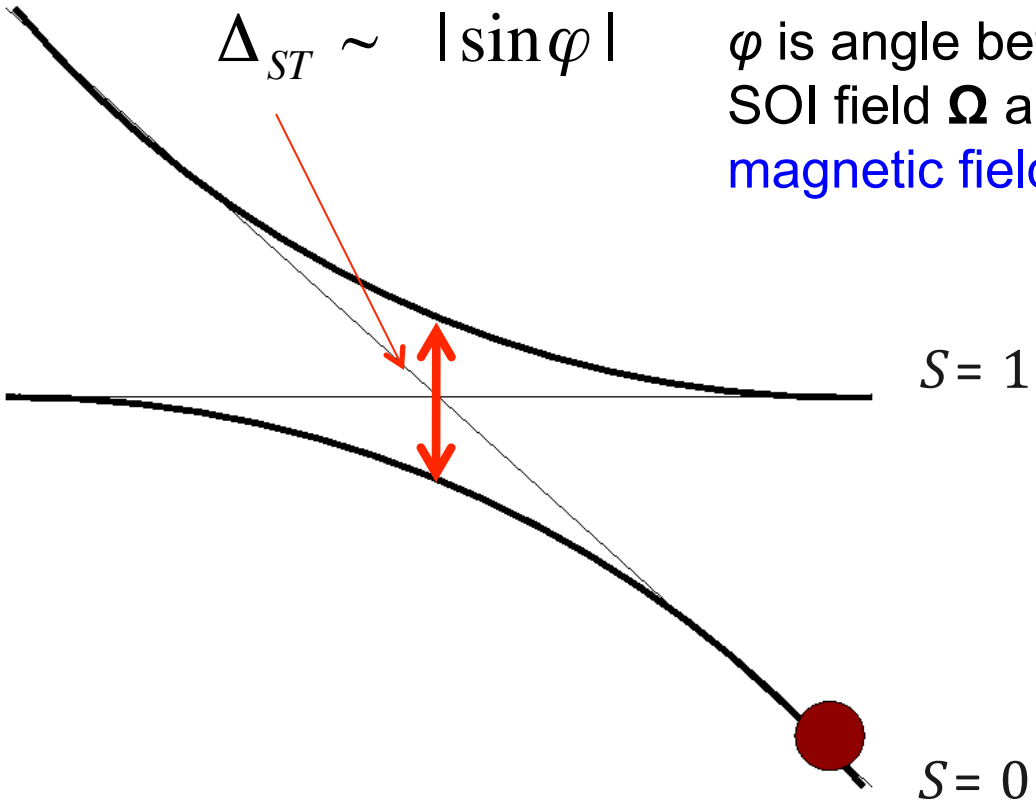
hyperfine interactions

mixing angle of (0,2)S and (1,1)S singlets depends on U,V, and t



Control of the splitting via magnetic field ***B*** (strength and direction)

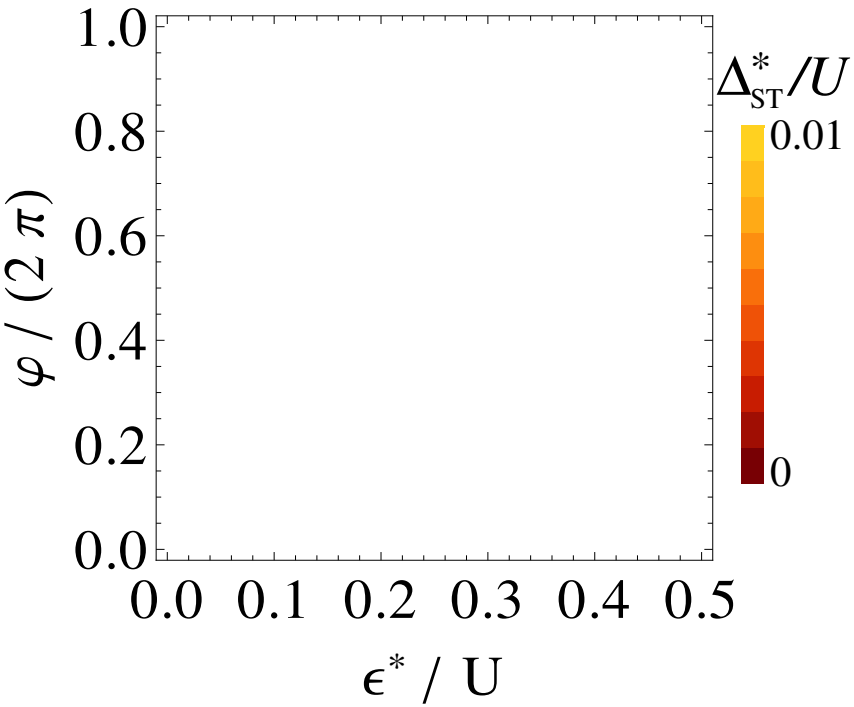
Level splitting Δ_{ST} is periodic with *B*-direction



φ is angle between SOI field **Ω** and magnetic field **B**

➔ direct experimental signature of spin orbit interaction !

Spin-orbit coupling constants from Δ_{ST}



$$\Delta^*_{\mathrm{ST}} = \frac{4t}{3}\frac{l}{\Lambda_{\mathrm{SO}}}|\sin\varphi|, \quad (\delta\mathbf{b} = 0)$$

$$\frac{1}{\Lambda_{\mathrm{SO}}} = \sqrt{\left(\frac{\cos\theta}{\lambda_-}\right)^2 + \left(\frac{\sin\theta}{\lambda_+}\right)^2}$$

θ: orientation of double dot in crystal

$$\lambda_\pm = \hbar/[m^*(\beta \pm \alpha)]$$

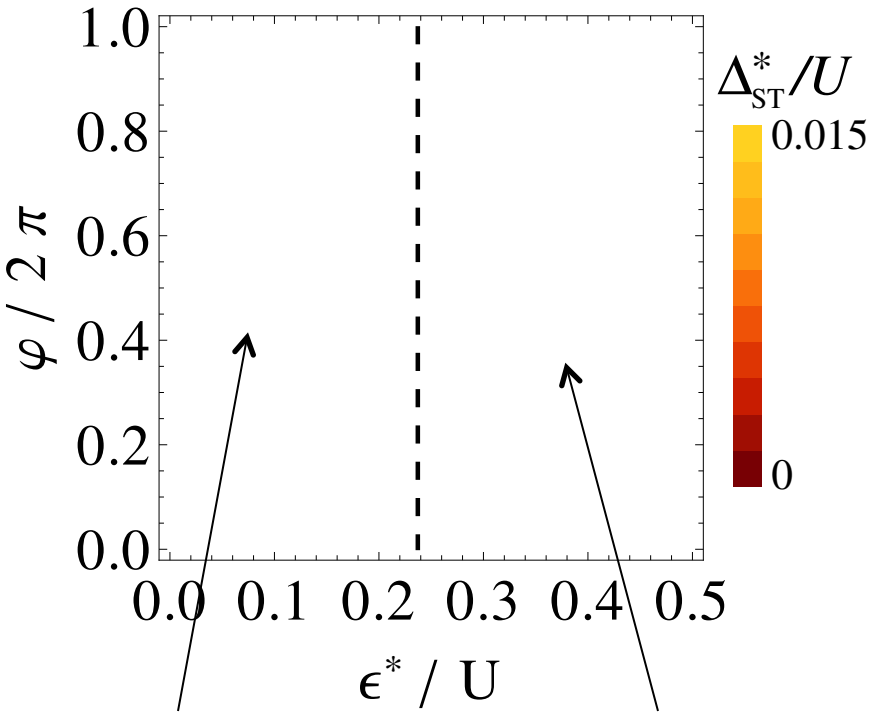
spin-orbit length

$$U = 1,\ t = 0.01, V_+ = 0.75,\ V_- = 0.74,\ \Omega = 0.005$$

Thus: Measurement of level splitting Δ_{ST} gives experimental access to spin orbit coupling constants α, β.

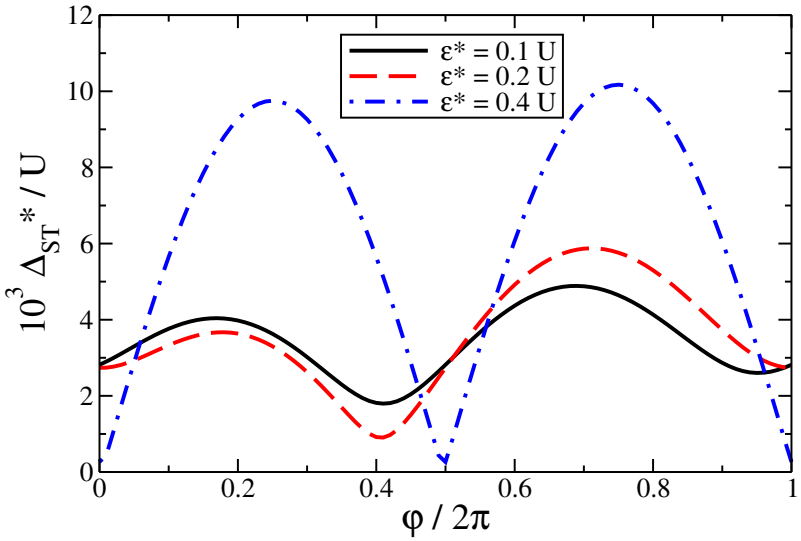
Splitting at Anti-Crossing

Stepanenko, Rudner, Halperin, DL, arXiv:1112.1644



Hyperfine regime

SOI regime

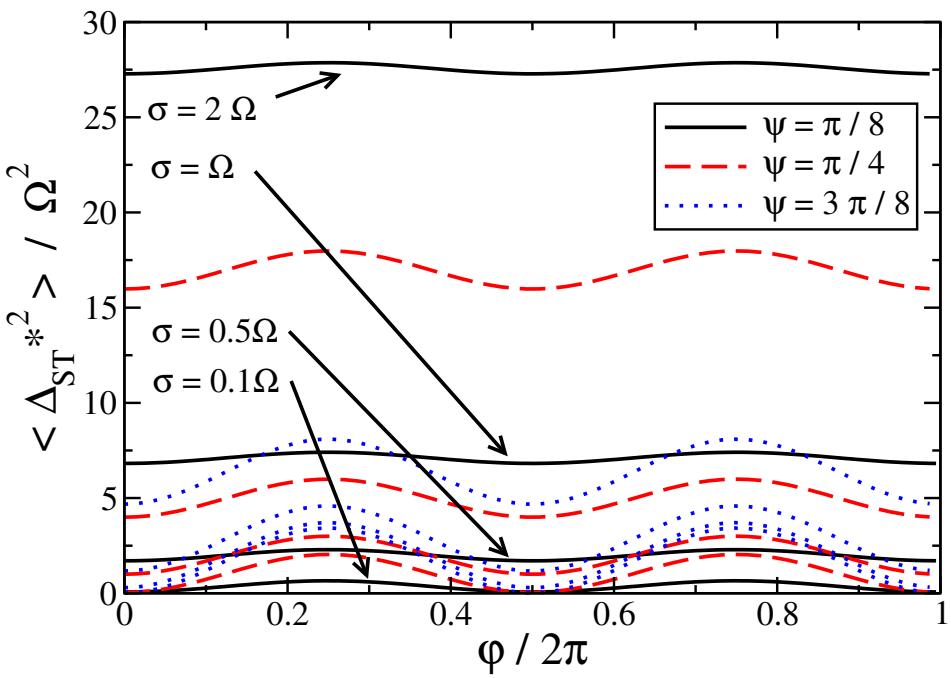


$$U = 1, \; t = 0.01,$$
$$V_+ = 0.75, \; V_- = 0.74, \; \Omega = 0.005$$
$$\delta\mathbf{b} = (-0.0006, 0.0008, 0.0012)$$

Thus: Angular dependence reveals the strengths of **both** spin-orbit and hyperfine interactions.

Hyperfine coupling from the **variance** of Δ_{ST}

Assume Gaussian distribution of hyperfine fields $\delta\boldsymbol{b}$ with zero mean and variance σ:



Weaker hyperfine fields (larger ψ) gives more pronounced oscillations

Note: large hyperfine variance σ smears out angular dependence!

Switching the spin transfer Δm

$$v_\vartheta = v_{\rm SO} + e^{i\vartheta} v_{\rm HF}$$

Rudner and Levitov, PRB (2010)

$$v_{\rm SO} = |\Omega \sin\varphi \sin\psi|$$

$$v_{\rm HF} = |\cos\psi| \sqrt{(\delta\mathbf{b}\cdot\mathbf{e}')^2 + \delta b_y^2}$$

Stepanenko et al. (2011)

I. Go to spin-orbit dominated regime: $\langle \Delta m \rangle = 0$

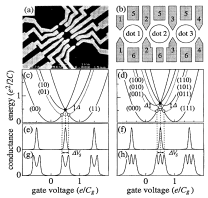
By applying strong magnetic field in the plane of the dot and perpendicular to SOI vector **Ω**

II. Switch to the hyperfine dominated regime: $\langle \Delta m \rangle = 1$

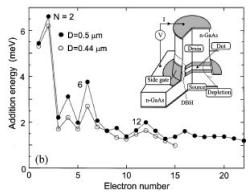
By reducing magnetic field to increase (1,1)S component of the crossing state

If SOI still dominates, rotate the magnetic field to point along ***Ω***.

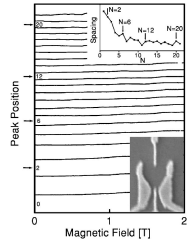
GaAs quantum dots as a tunable artificial atom



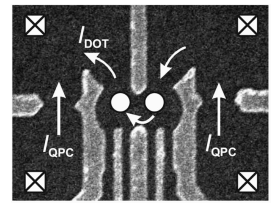
Westervelt
Gossard 1995



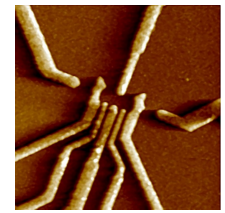
Kouwenhoven
Tarucha 1996



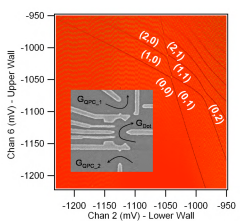
Sachrajda
2000



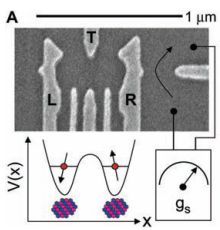
Kouwenhoven
Tarucha 2003/11



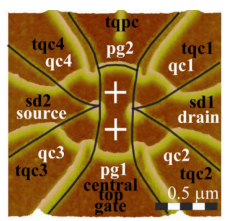
Vandersypen,
Koppen, 2003



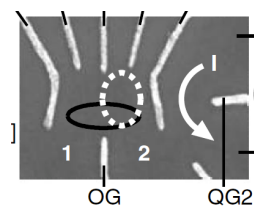
Marcus 2004



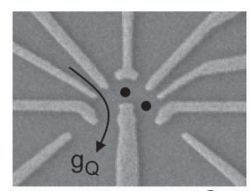
Petta, Marcus,
Yacoby 2005



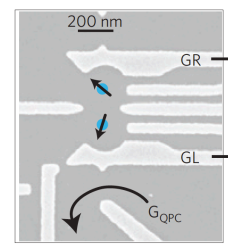
Ensslin, Ihn
2006



Zumbuhl,
Kastner
2008

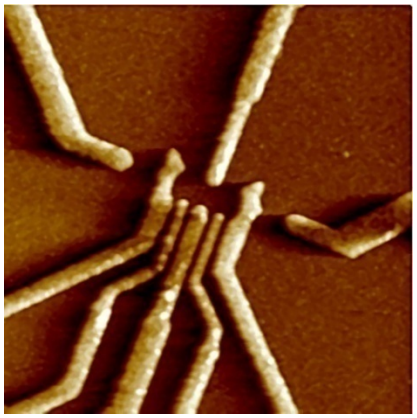


Petta 2010



Bluhm,
Yacoby 2010

Spin qubits in GaAs dots – present status



See also Hanson et al., Rev. Mod. Phys. 2007

All-electrical control and read-out achieved

Initialization 1-electron, low T, high B_0
duration ~ 5 T_1; 99% fidelity ?

Read-out via spin-charge conversion
duration ~ 100 μs; 82-97% fidelity

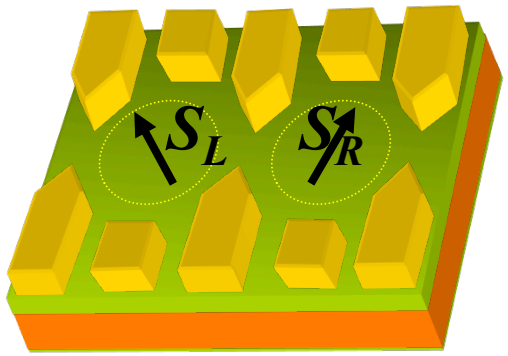
1-qubit gate electron spin resonance
gate duration ~ 25 ns; observed 8-50 periods

2-qubit gate exchange interaction
gate duration ~ 0.2 ns; observed 3 periods

Energy relaxation
T_1~ 1 sec

Phase coherence
T_2^* ~ 10-100 ns
T_2 > 1-270 μs

Spin-Qubits from Electrons



simplest spin-qubit:
spin-1/2 of 1 electron $|0\rangle = \uparrow,\ |1\rangle = \downarrow$

Many more choices for spin qubits:

- 'exchange-only qubits' DiVincenzo *et al.,* 2000
  3 electrons: $|0\rangle = S\uparrow,\ |1\rangle = T_+\downarrow - T_0\uparrow$
- 'singlet-triplet' qubits Levy 2002, Taylor *et al.,* 2005
  2 electrons: $|0\rangle = S,\ |1\rangle = T_0$
- 'spin-cluster qubits' Meier, Levy & DL, `03
  N electrons: AF spin chains, ladders, clusters,...
- `spin-orbit qubits‘ Golovach, Borhani &DL, `07, Kouwenhoven *et al.*, `11
- molecular magnets Leuenberger & DL, ’01; Affronte *et al.*, ’06,
  Lehmann *et al.*, ’07; Trif *et al.*, ’08, ’10

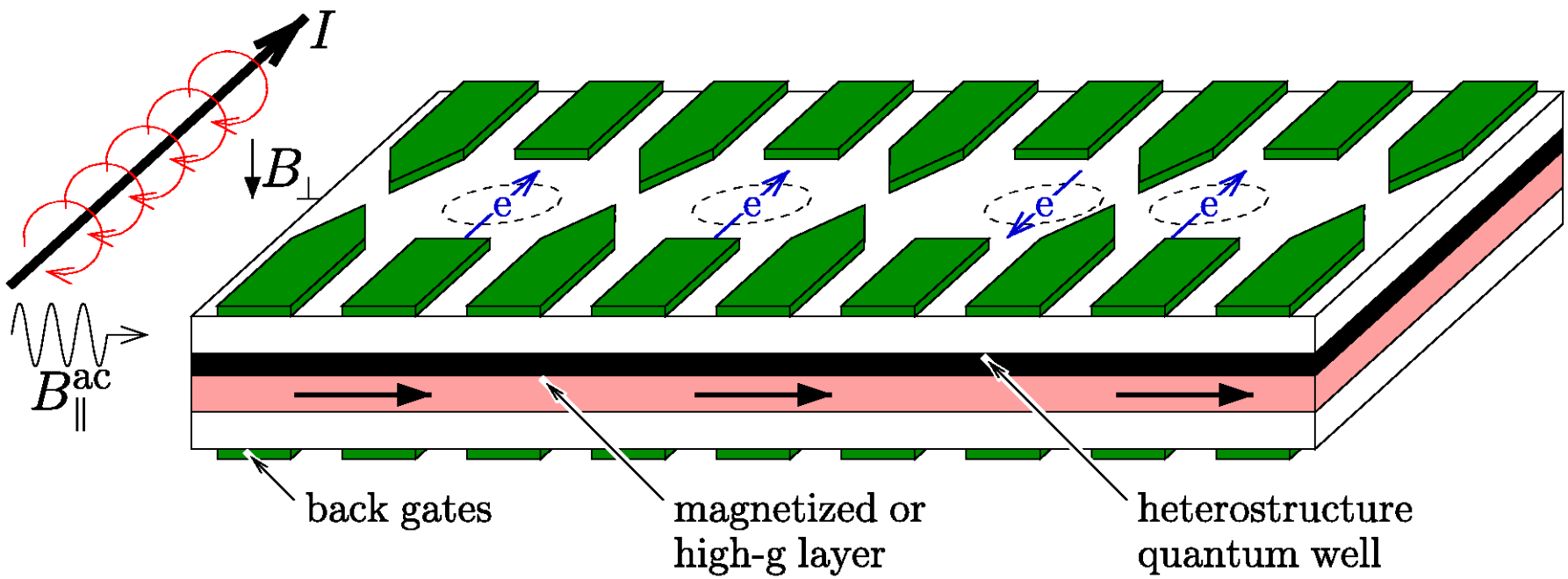
Outline

- spin qubits and quantum dots: GaAs and others
- long-distance spin-spin coupling (floating gates)

  ➔ **scalable 2D architecture**
- (Exotic) Bound states in CDW-wires as quantum dots

Scaling-up of spin qubits ➔ quantum dot array

DL & DiVincenzo, PRA **57** (1998) 120



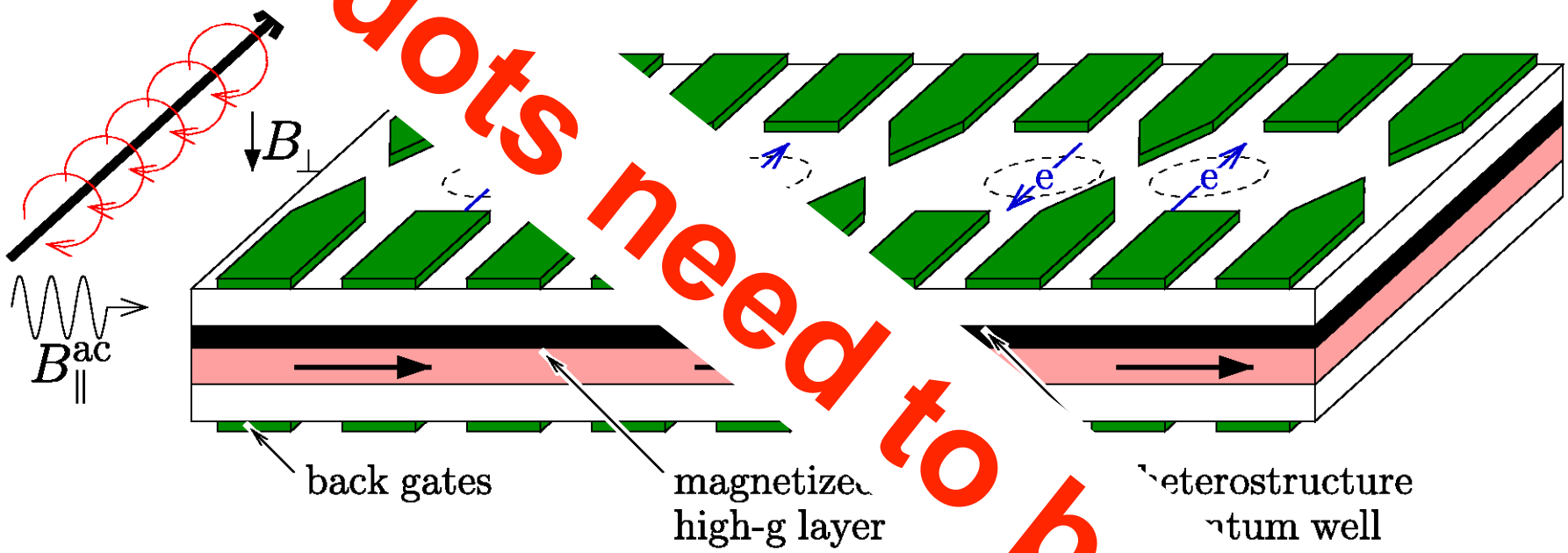
$$H = \sum_{\langle ij \rangle} J_{ij}(t)\mathbf{S}_i \cdot \mathbf{S}_j + \sum_i (g_i \mu_B \mathbf{B}_i)(t) \cdot \mathbf{S}_i$$

n.n. exchange local Zeeman

ling-up of spin qubits ➔ quantum dot array

DL & DiVincenzo, PRA **57** (1998) 120

But: dots need to be close



$$H = \sum_{\langle ij \rangle} J_{ij}(t)\,\mathbf{S}_i \cdot \mathbf{S}_j + \sum_i (g_i \quad (t)) \cdot \mathbf{S}_i$$

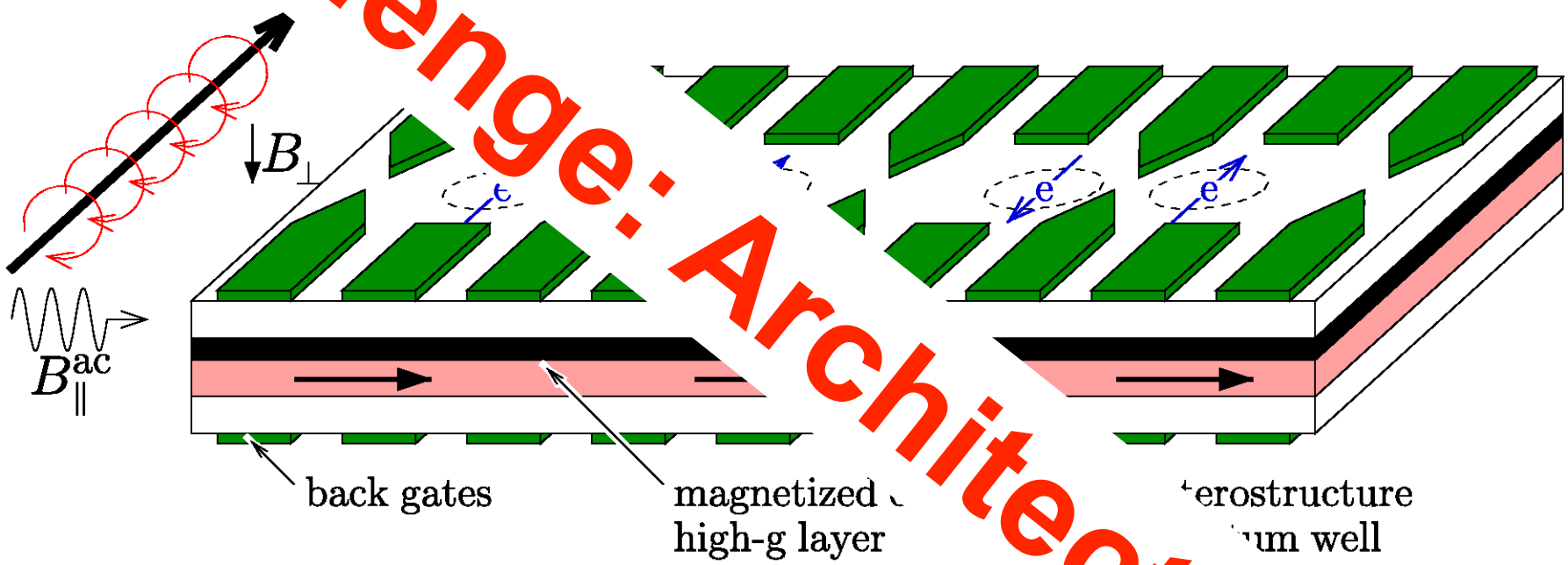
n.n. exchange

local Zee

Challenge: Architecture !?

ing-up of spin qubits ➔ quantum dot array

& DiVincenzo, PRA **57** (1998) 120



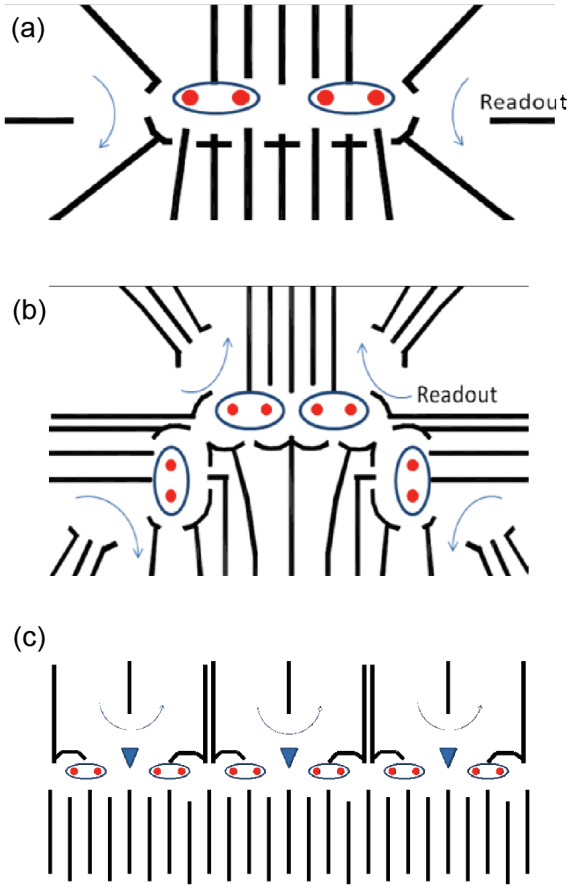
$$H = \sum_{\langle ij \rangle} J_{ij}(t)\mathbf{S}_i \cdot \mathbf{S}_j + \sum_i (g_i \mu \ldots) \cdot \mathbf{S}_i$$

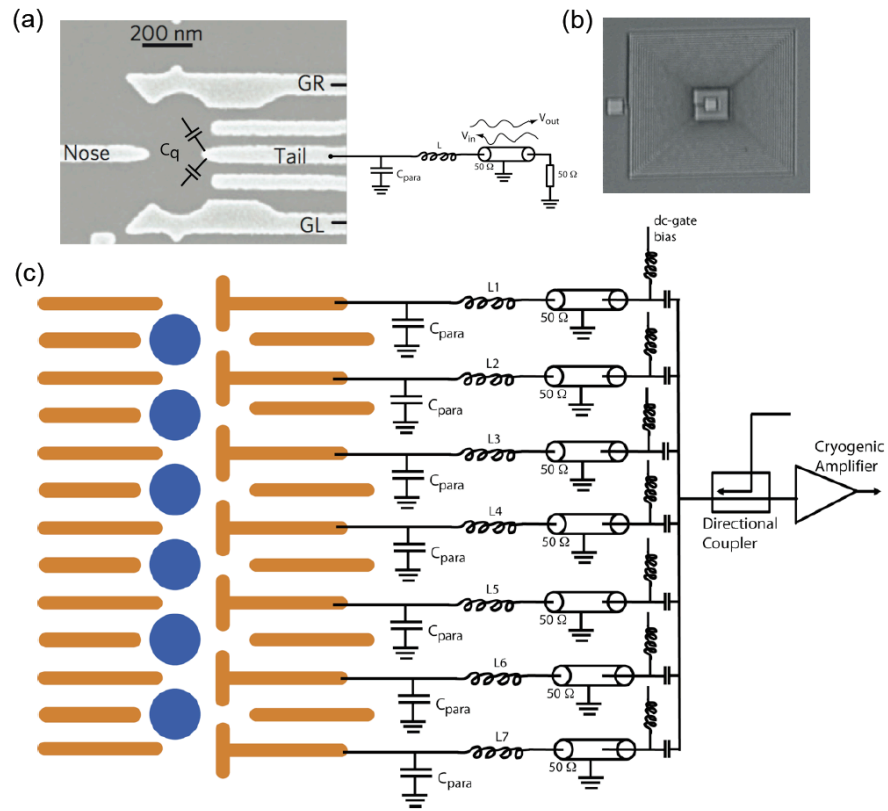
n.n. exchange

local Zeem

‘Blueprint’ for multi-qubit processors

IARPA consortium (Harvard, Basel, Delft, Maryland, Tokyo), since 2010

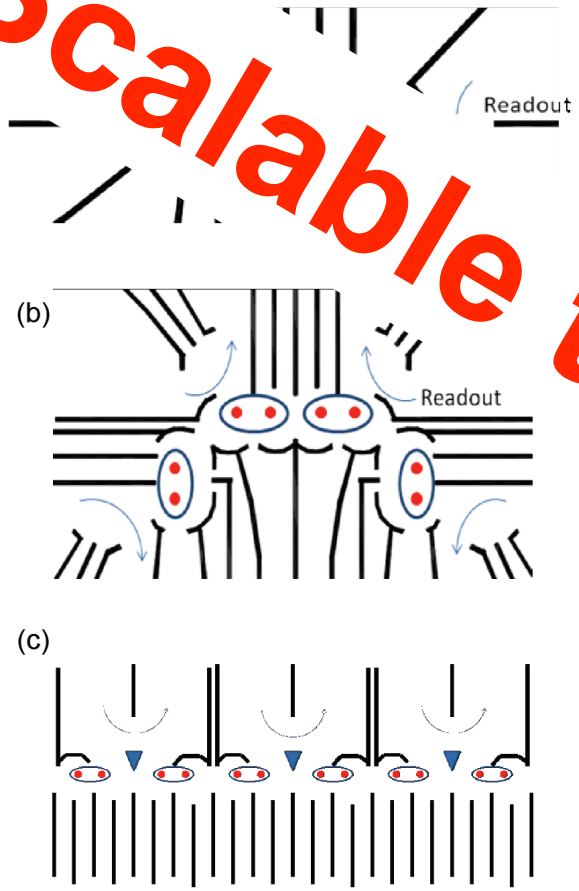


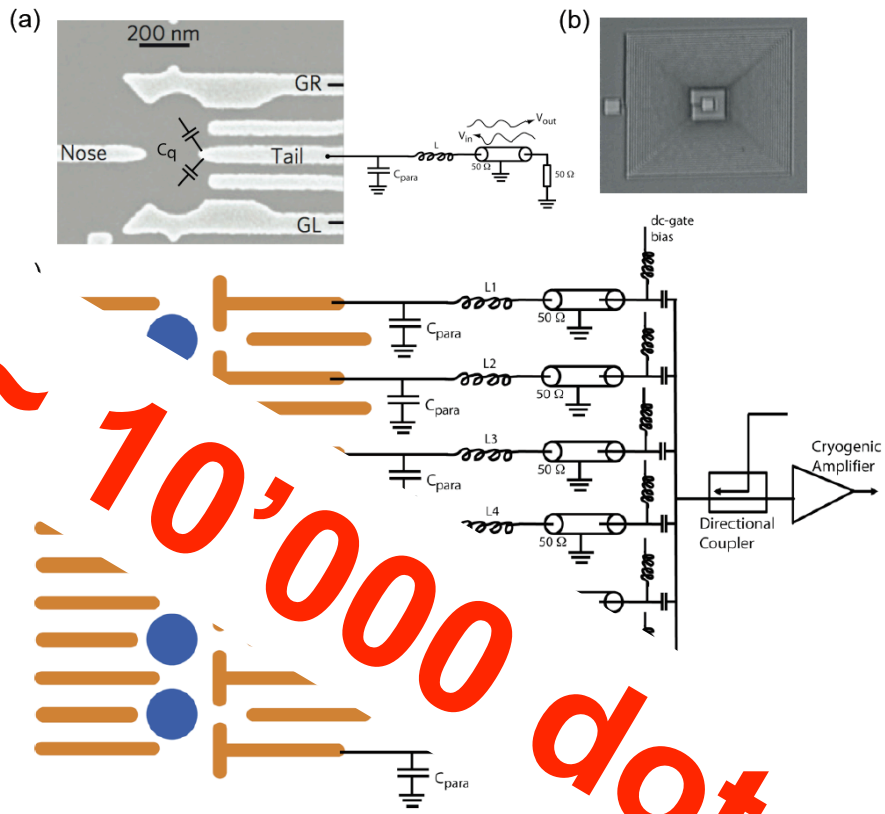


‘Blueprint’ for multi-qubit processors

IARPA consortium (Harvard, Basel, Delft, Maryland, Tokyo), since 2010

Scalable to ~ 10’000 dots?



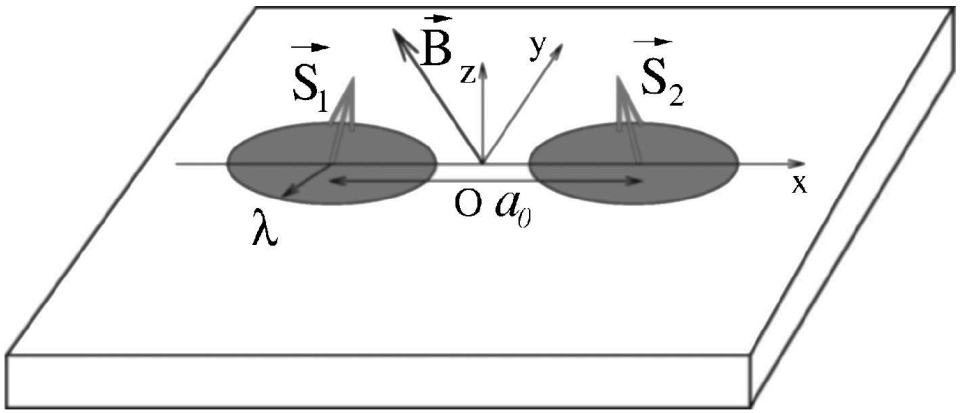


Effective Exchange without Tunneling

Trif, Golovach, DL, Phys. Rev. B 75, 085307 (2007)

Ingredients: Direct Coulomb interaction, spin orbit interaction, and magnetic field,

but no tunneling (overlap) between the dots!



$$H_s^{eff} = J_{eff}(\sigma_1^+\sigma_2^- + \sigma_2^+\sigma_1^-)$$

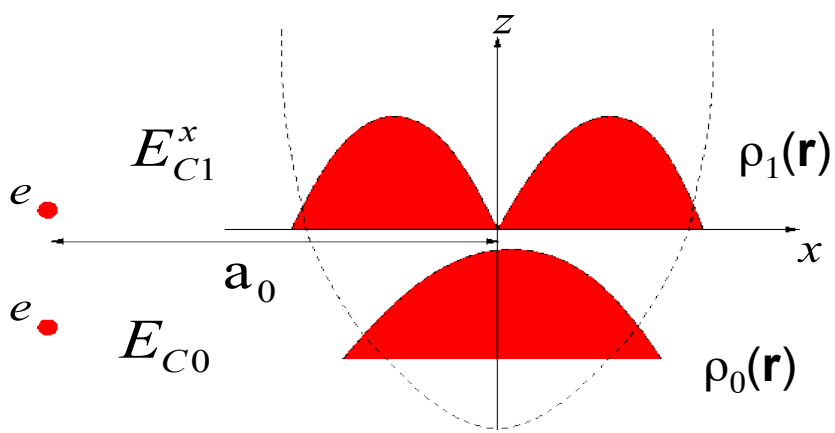
→ universal gate
Imamoglu et al., PRL 1999

Nanowire dots (1D): Flindt, Sorensen, and Flensberg, PRL **97**, 240501 (2006)
Vertical dots: Zhao, Zhong, Zhu, and C. P. Sun, PRB **74,** 075307 (2006)

$$J_{eff} = E_Z \frac{E_Z}{2\hbar\omega_0} \frac{\lambda}{a_B} \frac{\lambda^2}{\lambda_{SO}^2} G\left(\frac{a_0}{\lambda}, \theta, \varphi\right)$$

$$G = \left(\Delta E_C^x + \Delta E_C^y\right)\left(1 - \sin^2\theta \sin^2\varphi\right)$$

$$\Delta E_C^{x,y} = \int d\mathbf{r} \frac{\rho_0(\mathbf{r})\left(1 - r_{x,y}^2\right)}{\kappa\sqrt{y^2 + \left(x + a_0/\lambda\right)^2}}$$



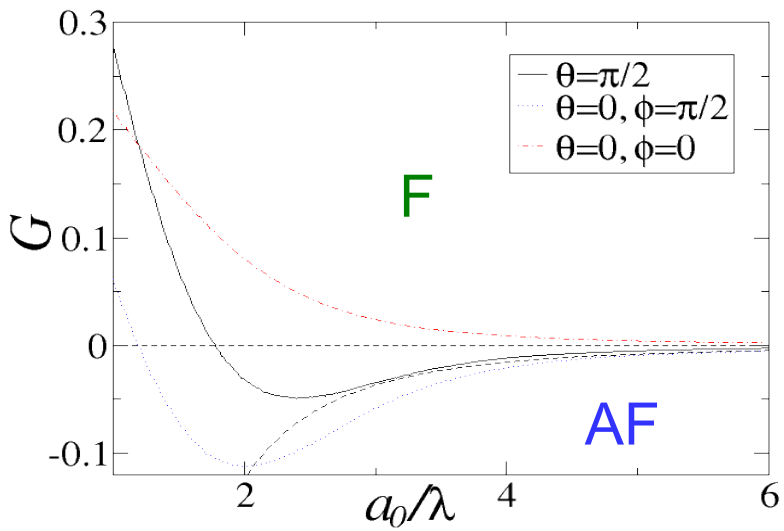
difference between electrostatic energies in the GS and 1st ES:

$$E_{C0} - E_{C1}^x = \Delta E_C^x$$
$$(x \to y)$$

$$J_{eff} = E_Z \frac{E_Z}{2\hbar\omega_0} \frac{\lambda}{a_B} \frac{\lambda^2}{\lambda_{SO}^2} G\left(\frac{a_0}{\lambda}, \theta, \varphi\right)$$

$$G = \left(\Delta E_C^x + \Delta E_C^y\right)\left(1 - \sin^2\theta \sin^2\varphi\right)$$

$$\Delta E_C^{x,y} = \int d\mathbf{r} \frac{\rho_0(\mathbf{r})\left(1 - r_{x,y}^2\right)}{\kappa\sqrt{y^2 + \left(x + a_0/\lambda\right)^2}}$$



Numerical estimates (strong Coulomb)

$$J_{eff} \sim E_Z \frac{E_Z}{\hbar\omega_0}\left(\frac{\lambda}{\lambda_{SO}}\right)^2 I$$

GaAs QDs:

$B = 9\mathrm{T}\left(E_Z \approx 0.3\,\mathrm{meV}\right)$

$\hbar\omega_0 \approx 0.5\,\mathrm{meV} \quad \rightarrow \quad \lambda / a_B \sim 5$

$\lambda_{SO} \approx 3\cdot 10^{-6}\,\mathrm{m}, \ \left(\lambda / \lambda_{SO} \approx 0.3\cdot 10^{-1}\right)$

$a_0 / \lambda \cong 2$

$$J_{eff} \approx 10^{-4}\,\mathrm{meV} \ \sim [5ns]^{-1}$$

InAs QDs:

$B = 30\ \mathrm{mT}\ \left(E_Z \approx 0.3\ \mathrm{meV}\right)$

$\hbar\omega_0 \approx 0.5\mathrm{meV} \quad \rightarrow \quad \lambda / a_B \sim 5$

$\lambda_{SO} \approx 10^{-7}\,\mathrm{m}, \quad \left(\lambda / \lambda_{SO} \approx 0.5\right)$

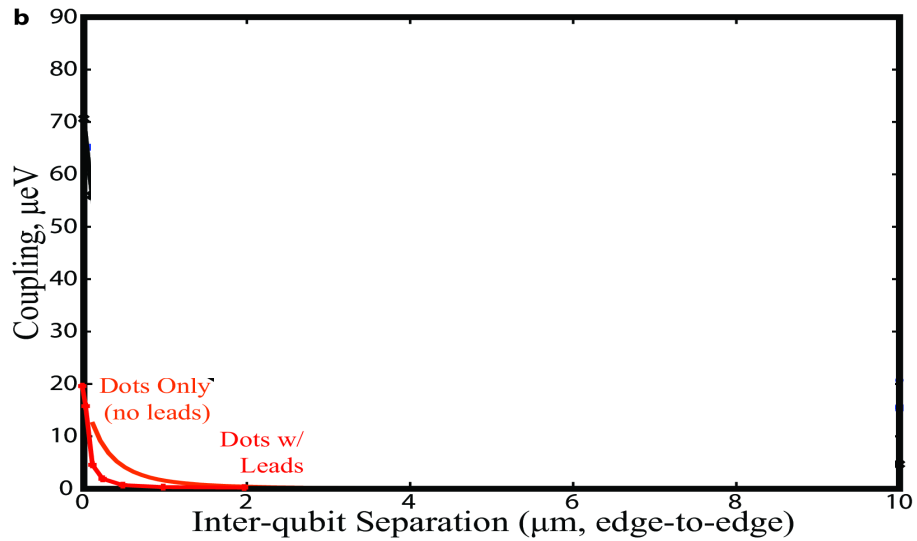
$a_0 / \lambda \cong 2$

$$J_{eff} \approx 10^{-2}\,\mathrm{meV} \ \sim [50ps]^{-1}$$

...of the same order as the hyperfine interaction A/√N between the electron and the N=10^5 nuclear spins in QD!

A Problem...

Coulomb interaction is screened due to 2DEG



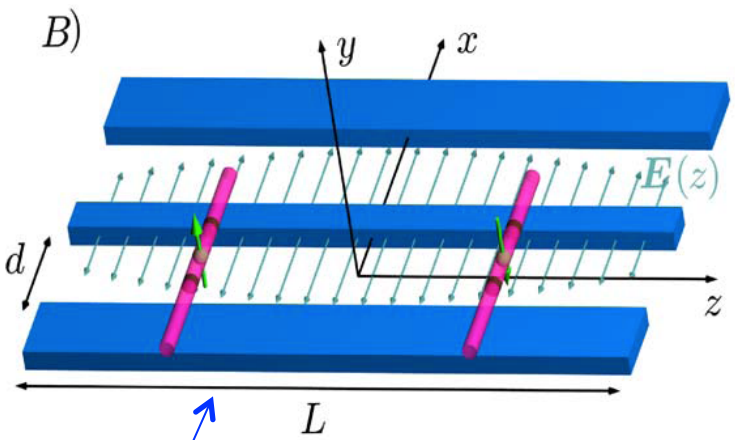
The resulting qubit-qubit coupling is effectively short-ranged!

Trifunovic, Day, Trif, Wootton, Abebe, Yacoby, and DL, arxiv: 1110.1342

Spins coupled to Stripline Photons

Trif, Golovach, DL, PRB 77, 045434 (2008)

nanowire with large spin orbit interaction: allows to couple spin to quantized electric field (photon)



- 1D-shaped QDs in InAs nanowires

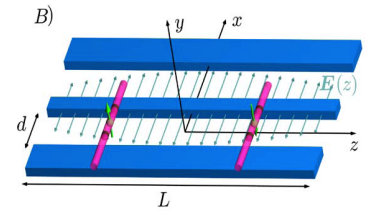
Fasth et al., Nano Lett. 5, 1487 (2005)

1D cavity: Wallraff et al. Nature 431,162 (2004)

Long-distance Spin-Spin Coupling

Spin1-photon + Spin2-photon ➔ Spin1-Spin2

$$H_{s-s} = \sum_{i=1,2} \tilde{E}^{i}_{iZ} \sigma^{i}_{Z} + J\left(\sigma^{1}_{+}\sigma^{2}_{-} + \sigma^{2}_{+}\sigma^{1}_{-}\right)$$



XY Spin-Spin Interaction – universal for quantum computing!

$J = \frac{\nu_{E,1}\nu_{E,2}}{2}\left(\frac{1}{\Delta_1} + \frac{1}{\Delta_2}\right)$: effective exchange coupling – ***long-range (millimeters!)***

effective Zeeman + Stark shift + Lamb shift: $\tilde{E}^{eff}_{iZ} = E^{eff}_{iZ} + \frac{\nu^2_{E,i}}{\Delta_i}\left(n + \frac{1}{2}\right)$

detuning of the spin from the cavity mode: $\Delta_i = E^{eff}_{Z} - \hbar\omega$

Trif, Golovach, and DL, PRB 77, 045434 (2008)

Some Estimates

Disk-shaped QDs

$$R = 50\,\text{nm}\left(\Delta E_0 \cong 5\,\text{meV}\right)$$
$$\lambda_{SO} \approx 100\,\text{nm},\ g \approx 2.5$$
$$L = 2\,\text{mm},\ d = 100\,\text{nm} \Rightarrow$$
$$E_{cav} \approx 100\,\text{V/m}, \quad \hbar\omega \approx 0.5\,\text{meV}$$
$$E_Z = 0.5\,\text{meV},\ \left(B \approx 1.75\,\text{T}\right)$$

$$\nu_E \approx 4\cdot 10^{-5}\ \text{meV}\ (\tau_{\text{Rabi}} \approx 200\,\text{ns})$$
$$J \approx 10^{-6}\ \text{meV}\ (\tau_{\text{switching}} \approx 500\,\text{ns})$$

1D-shaped QDs

$$l = 40\,\text{nm}\left(\Delta E_0 \cong 2\,\text{meV}\right)$$
$$\lambda_{SO} \approx 100\,\text{nm},\ g \approx 10$$
$$L = 2\,\text{mm},\ d = 100\,\text{nm} \Rightarrow$$
$$E_{cav} \approx 100\,\text{V/m}, \quad \hbar\omega \approx 0.5\,\text{meV}$$
$$E_Z = 0.5\,\text{meV},\ \left(B \approx 0.45\,\text{T}\right)$$

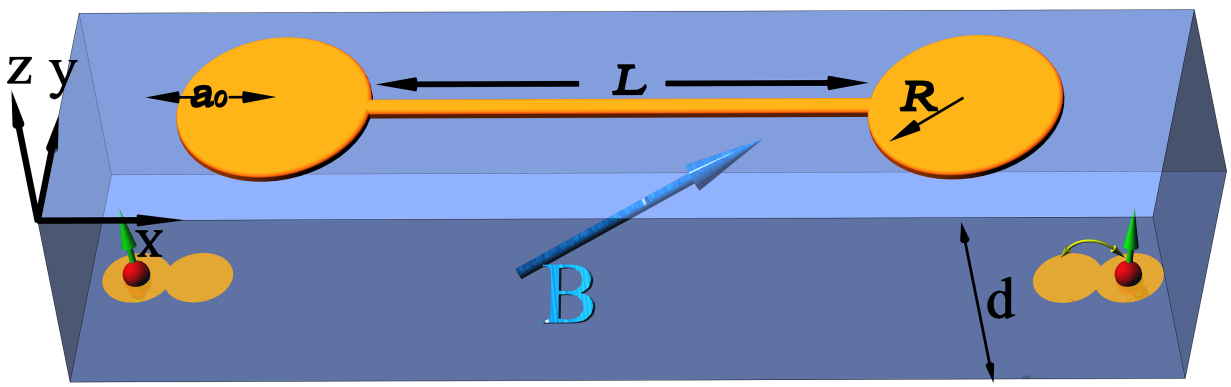
$$\nu_E \approx 4\cdot 10^{-4}\ \text{meV}\ (\tau_{\text{Rabi}} \approx 2\,\text{ns})$$
$$J \approx 4\cdot 10^{-5}\ \text{meV}\ (\tau_{\text{switching}} \approx 20\,\text{ns})$$

ν_E can be changed by changing the g – factor (e.g. by using top-gates)

Direct coupling of B-field in the cavity: $\nu_B = g\mu_B B_{cav} \approx 3\cdot 10^{-8}\ \text{meV} \approx 10^{-3}\nu_E$!

Long-distance spin-spin coupling via floating gates

Trifunovic, Day, Trif, Wootton, Abebe, Yacoby, DL, arxiv: 1110.1342 (PRX, in print)

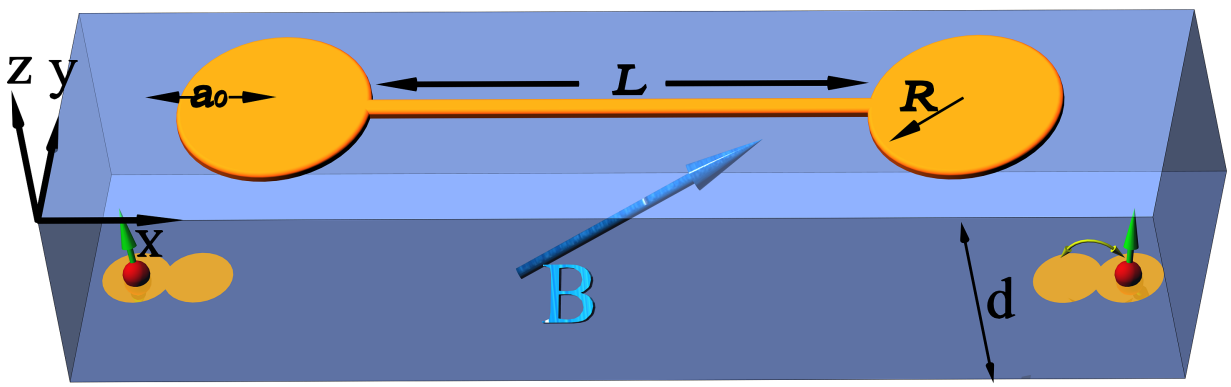


Electrostatics + spin orbit interaction ➔ effective exchange:

$$H_{s-s} = J_x \sigma_1^x \sigma_2^x + J_y \sigma_1^y \sigma_2^y$$

Long-distance spin-spin coupling via floating gates

Trifunovic, Day, Trif, Wootton, Abebe, Yacoby, DL, arxiv: 1110.1342 (PRX, in print)



Electrostatics + spin orbit interaction ➔ effective exchange:

$$J \simeq \frac{\pi \alpha_q \alpha_C}{4} \left(\frac{\partial q_{ind}}{\partial \tilde{x}} \right)^2_{\boldsymbol{r}=0} \left(\frac{E_Z}{\omega_x} \right)^2 \left(\frac{\lambda}{\lambda_{SO}} \right)^2 \hbar \omega_x \sim \text{1-100 μeV}$$

$$\tau_{\text{switching}} = \hbar / J \approx 1\text{ns} \ - \ 10\text{ps}$$

Qubit-Qubit coupling via Schrieffer-Wolff trafo

Trifunovic *et al.*, arxiv: 1110.1342 (PRX, in print)

$$H = V + \sum_{i=1,2} H^i_{qubit}$$

$$H_{QD} = H_0 + H_Z + H_{SO},$$

$$H_{DQD} = J\, \boldsymbol{S}_1 \cdot \boldsymbol{S}_2 + H^1_Z + H^2_Z$$

perturbation: $$H_{SO} = \alpha(p_x\sigma_y - p_y\sigma_x) + \beta(-p_x\sigma_x + p_y\sigma_y)$$

SW-trafo *): $$UHU^+ = H_d + \frac{i}{2}\int_0^\infty dt e^{-\eta t}[H_{SO}(t), H_{SO}] + O(H^4_{SO})$$

➔ $$H_{S-S} = J_{12}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\gamma})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{\gamma})$$

$$J_{12} = \frac{m^* \omega^2_{x,12} E^2_Z}{2(\omega^2_x - E^2_Z)^2}, \qquad \omega^2_{x,12} = \pi\alpha_q\alpha_C \left(\frac{\partial q_{ind}}{\partial \tilde{x}}\right)^2_{\boldsymbol{r}=0} \omega^2_x$$

*) Bravyi, DiVincenzo, and Loss, Annals of Physics 326, 2793-2826 (2011)

CNOT Gate

Trifunovic, Day, Trif, Wootton, Abebe, Yacoby, DL, arxiv: 1110.1342 (PRX, in print)

Problem: Hamiltonian does not commute with Zeeman terms:

$$H_{S-S} = J_{12}(\Gamma_1 - i\Gamma_2\sigma_z^1)\boxed{(\sigma_x^1\sigma_x^2 - \sigma_y^1\sigma_y^2)/2} + J_{12}|\gamma_{\mathbf{x}}|^{\mathbf{2}}(\sigma_x^1\sigma_x^2 + \sigma_y^1\sigma_y^2)/2.$$

But get good approximation for large B-field (perp. to 2DEG):

$$H_{S-S} \approx H'_{S-S} = \frac{J_{12}|\gamma_x|^2}{2}(\sigma_x^1\sigma_x^2 + \sigma_y^1\sigma_y^2),$$

$$H \approx H' = H'_{S-S} + E_z(\sigma_z^1 + \sigma_z^2)/2.$$

CNOT Gate & Fidelity

Trifunovic, Day, Trif, Wootton, Abebe, Yacoby, DL, arxiv: 1110.1342 (PRX, in print)

$$H_{S-S} \approx H'_{S-S} = \frac{J_{12}|\gamma_x|^2}{2}(\sigma_x^1\sigma_x^2 + \sigma_y^1\sigma_y^2),$$
$$H \approx H' = H'_{S-S} + E_z(\sigma_z^1 + \sigma_z^2)/2.$$

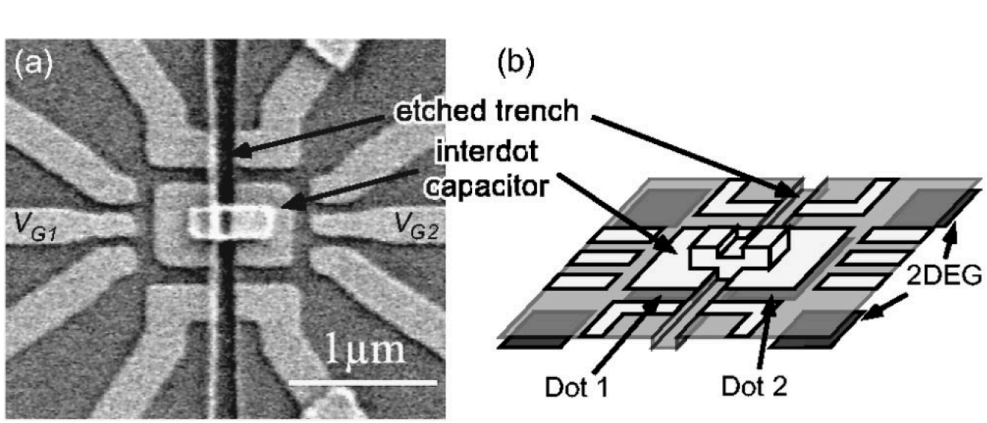
$$C = \sqrt{\sigma_z^1}\,\sqrt{\sigma_x^2}\,\mathcal{H}^1 e^{i(\sigma_z^1+\sigma_z^2)E_z t} e^{-iH't}$$
$$\sigma_x^1\, e^{i(\sigma_z^1+\sigma_z^2)E_z t} e^{-iH't}\, \sigma_x^1\, \mathcal{H}^1,$$

Fidelity: 99.993% for $J_{12}(\gamma_x)_x/E_Z = 0.01$

Strongly capacitively coupled quantum dots

I. H. Chan and R. M. Westervelt[a)]
Division of Engineering and Applied Sciences and Department of Physics, Harvard University, Cambridge, Massachusetts 02138

K. D. Maranowski[b)] and A. C. Gossard
Materials Department and Department of Electrical and Computer Engineering, University of California, Santa Barbara, California 93106



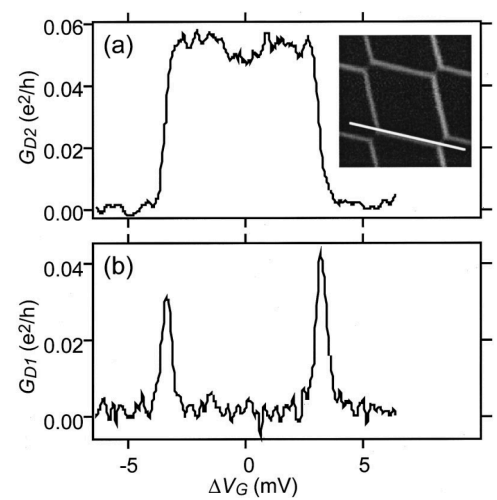
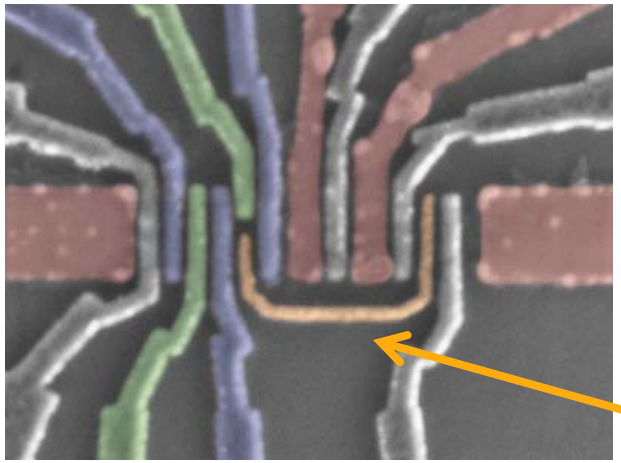


FIG. 3. Conductance (a) G_{D2} for dot 2 and (b) G_{D1} for dot 1 along the white line path in the inset. The addition of one electron to dot 1 switches dot 2 on or off a Coulomb blockade peak.

Double Quantum Dots in Nanotubes

Churchill,..., Marcus, *Nature Phys.* ***5****, 321 (2009)*



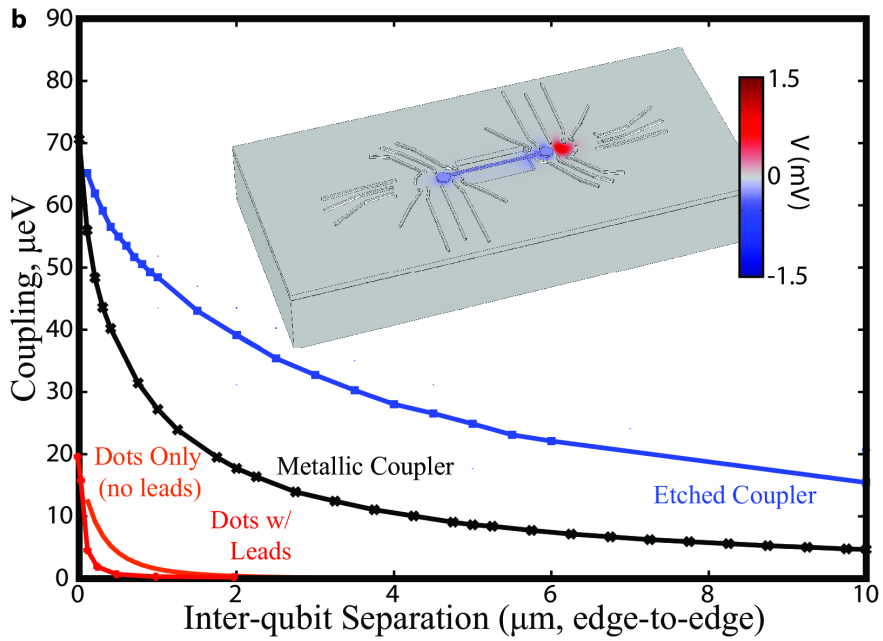
‘dog-bone’ gate for charge read-out

Note: Coulomb interaction V(r) matters!

Numerics

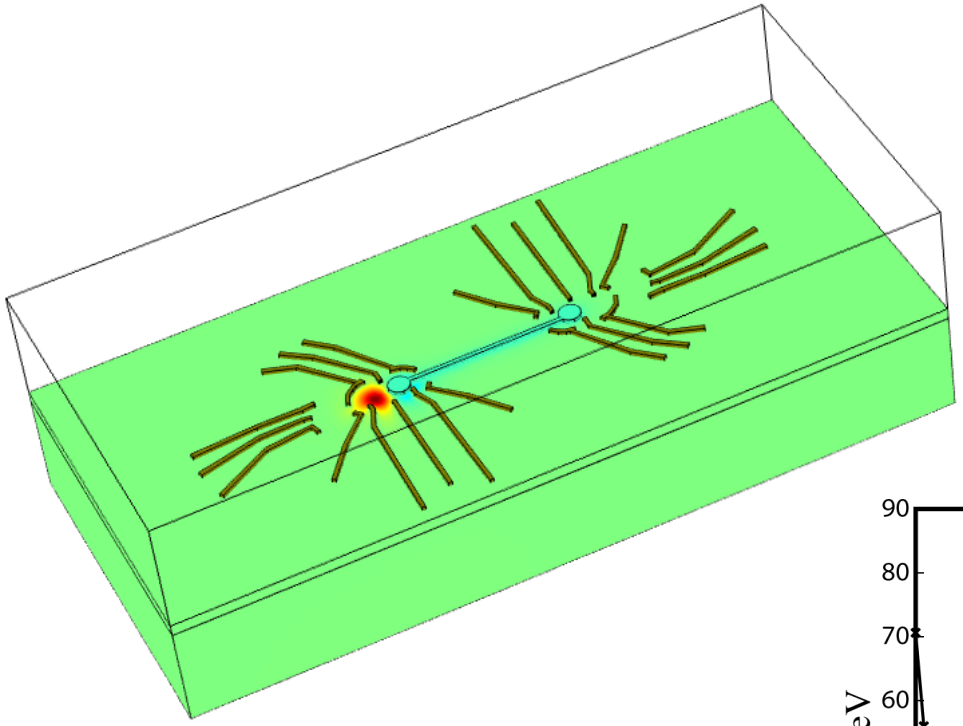
Trifunovic, Day, Trif, Wootton, Abebe, Yacoby, DL, arxiv: 1110.1342 (PRX, in print)

Long-range electrostatic coupling via metal or 2DEG channel

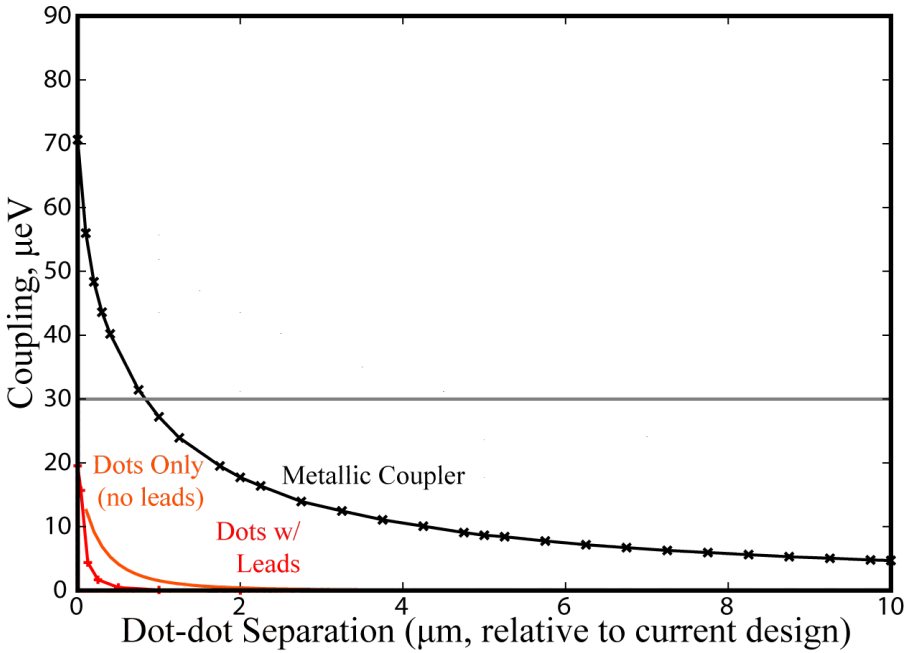


Note: gradient dV(r)/dr matters!

Metallic Electrostatic Couplers

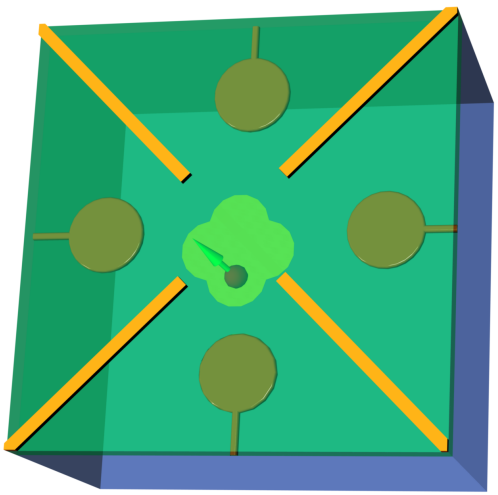


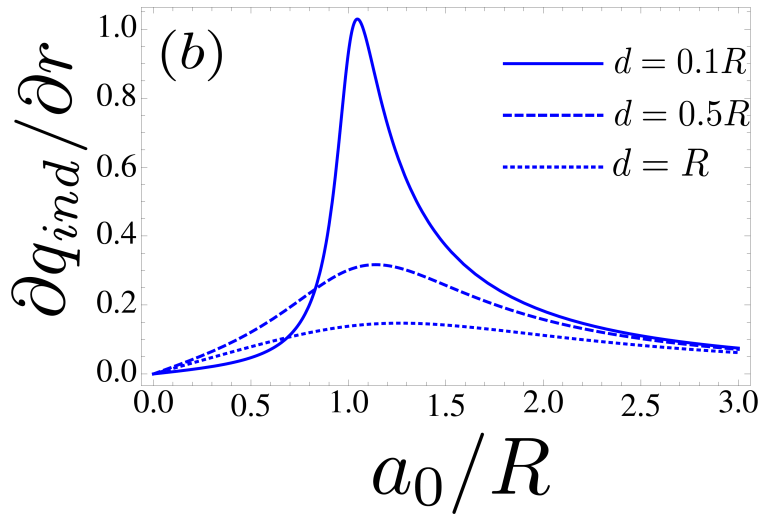
As the length of the gate increases, the coupling still drops rapidly due to shunt capacitance of the coupler to the 2DEG.



Switching the coupling on/off

Trifunovic et al., arxiv: 1110.1342



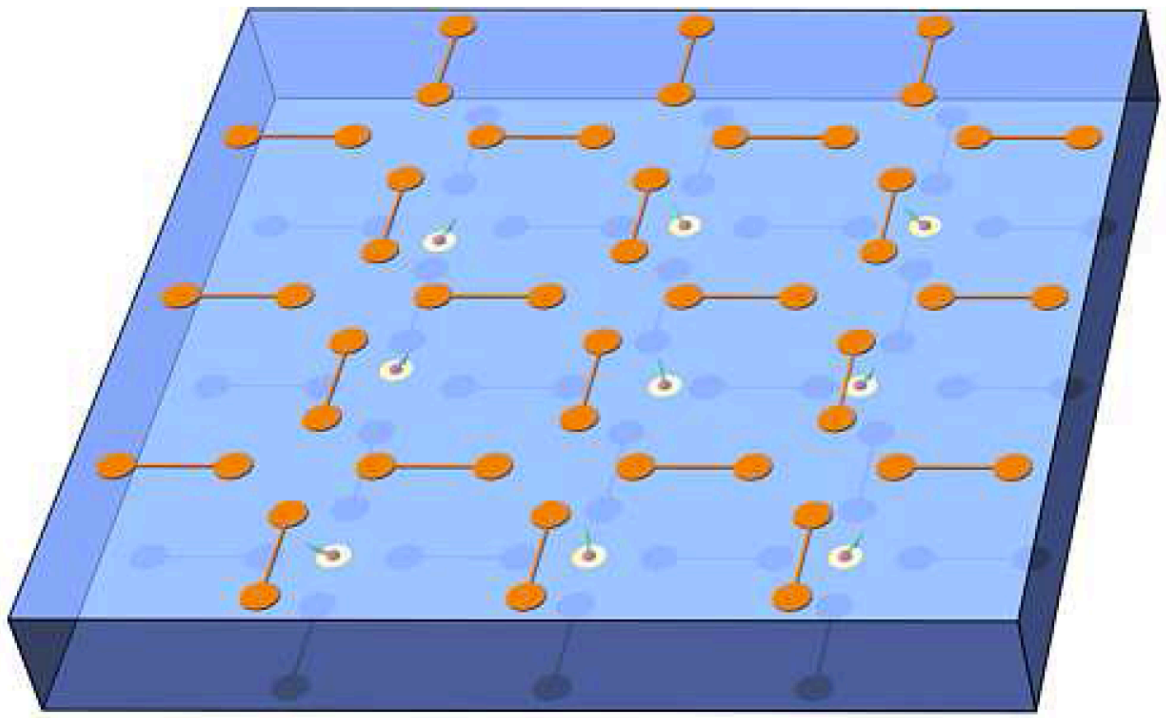


Control over the coupling is achieved by moving the electron to one of five possible spots

Note: in idle state the gate fluctuations (Nyquist noise) lead to standard but small spin decoherence

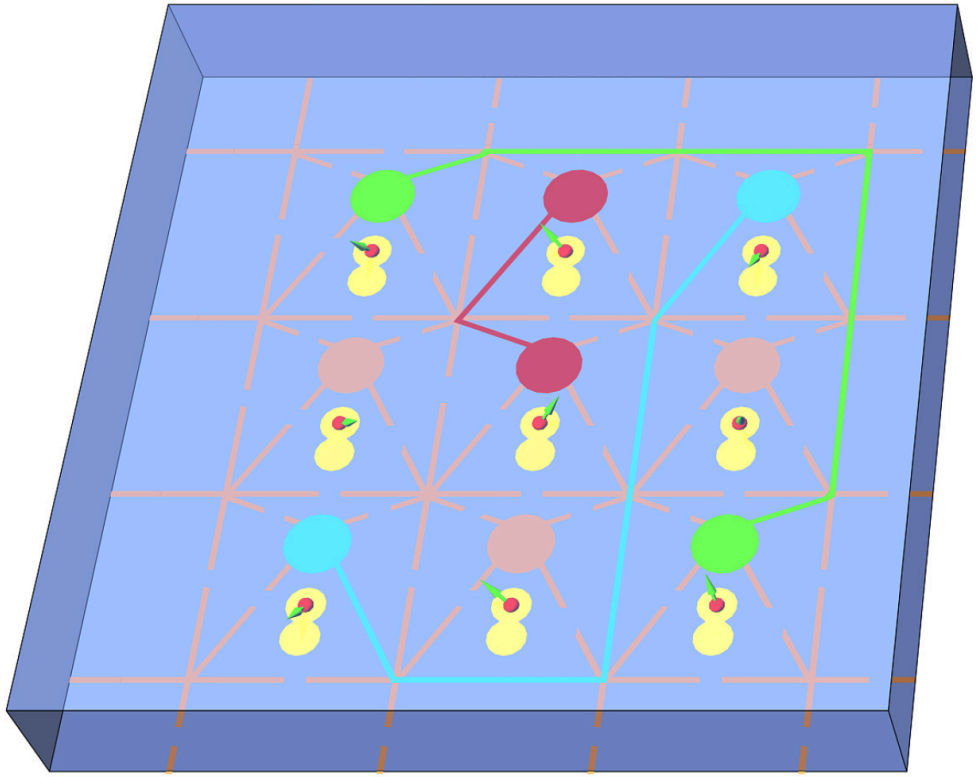
Metallic Electrostatic Coupler Network

Trifunovic et al., arxiv: 1110.1342



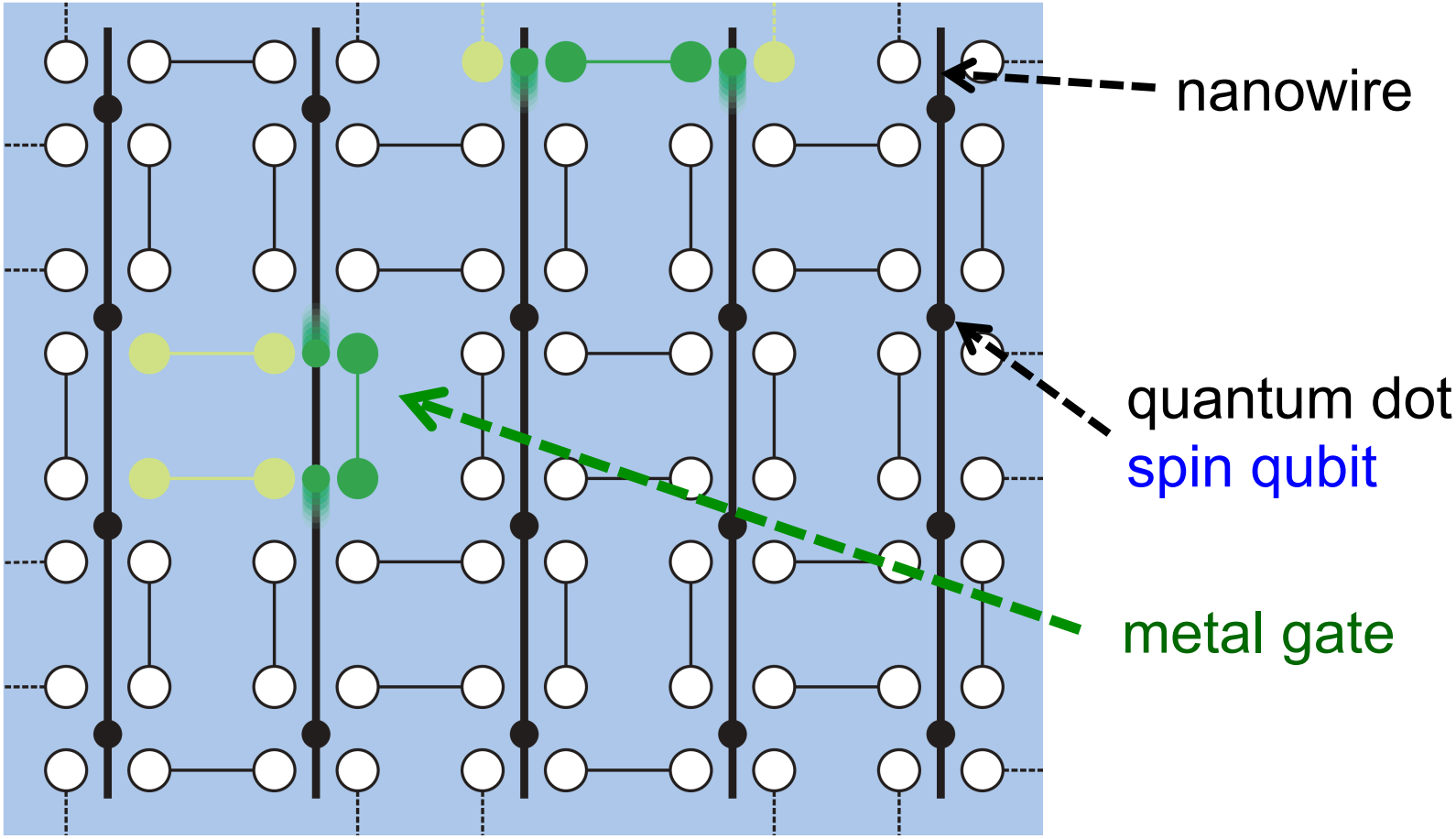
Etched Electrostatic Coupler Network

Trifunovic et al., arxiv: 1110.1342



Scalable Quantum Computer with Nanowires

Trifunovic et al., arxiv: 1110.1342



Semiconducting Nanowires

Various materials:
ZnO, InAs, InP, GaAs, AlAs, Ge, Si, SiGe, GaN, GaP, CdS,
…

Operate both in the
conduction band (CB) and valence band (VB)

Charge: *similar*
Spin: *very different*

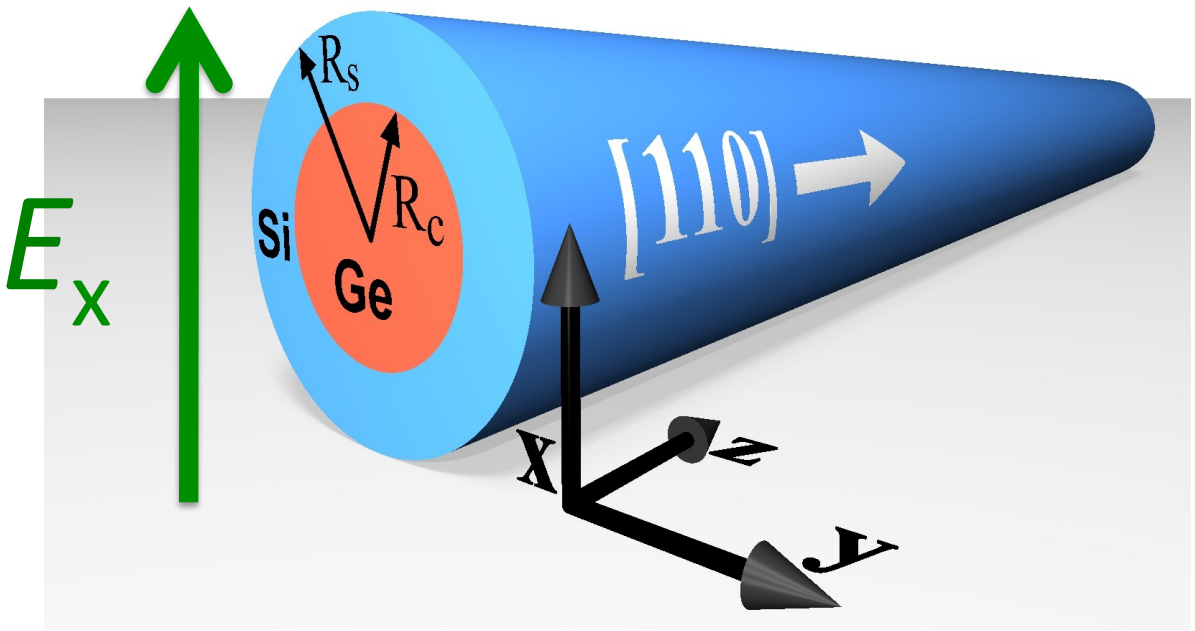
ELECTRONS ⟷ **HOLES**

Particularly characteristic for semiconductors

Holes turn out to be **advantageous in many aspects!**

Ge/Si Core/Shell Nanowires

Xiang *et al.*, Nature (2006); Hu *et al.*, Nat. Nano (2007); Hu *et al., preprint (2011)*



Nanowire
grown along [110]

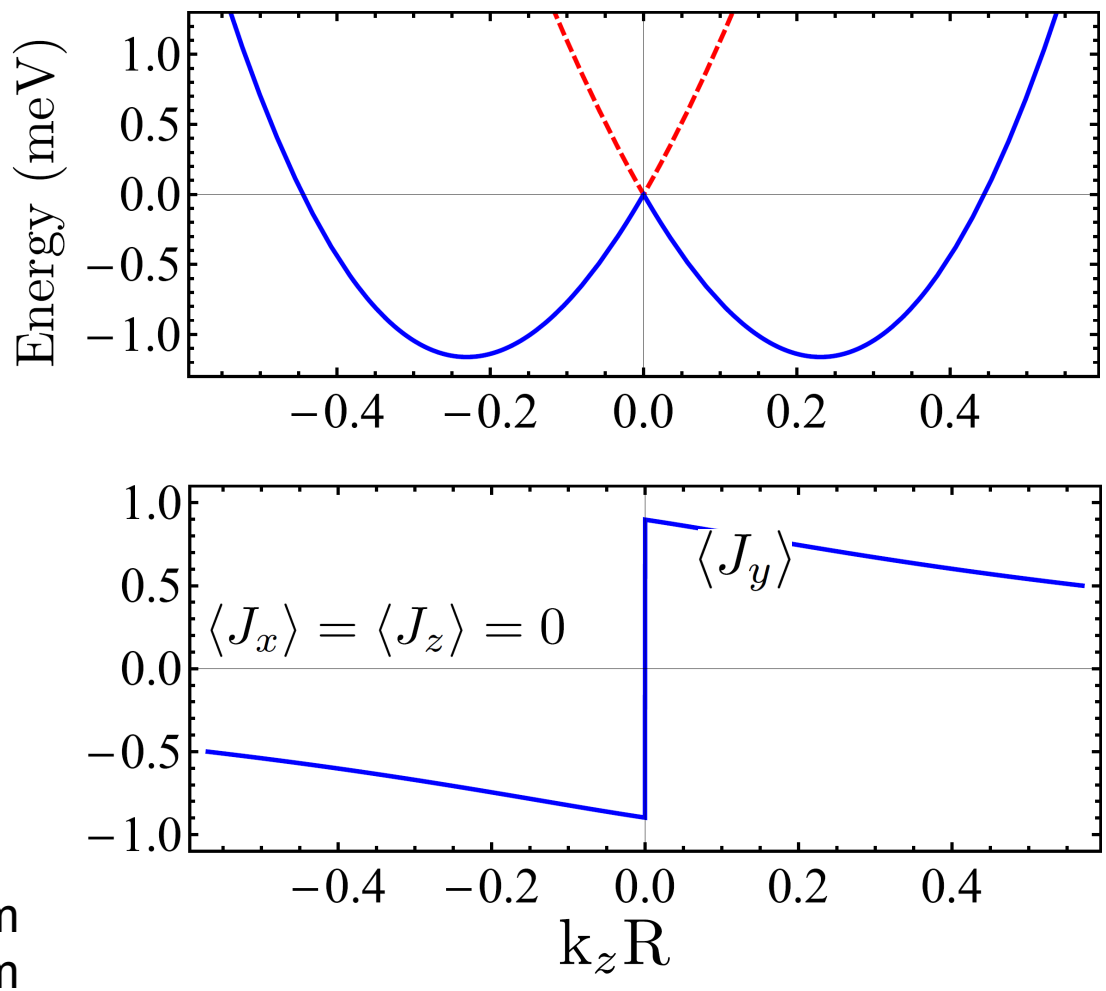
Large Ge/Si valence band offset of ~ 0.5 eV, narrow interfaces
→ **replace with hard wall at core radius $R_c \equiv R$**

Lauhon *et al.*, Nature (2002), Lu *et al.*, PNAS (2005)

Helical Hole States & Giant SOI

Kloeffel, Trif, and DL, Phys. Rev. B 84, 195314 (2011)

E-field along *x*: E_x = 6 V/μm ➔ strong Rashba SOI (~ 1-10 meV)



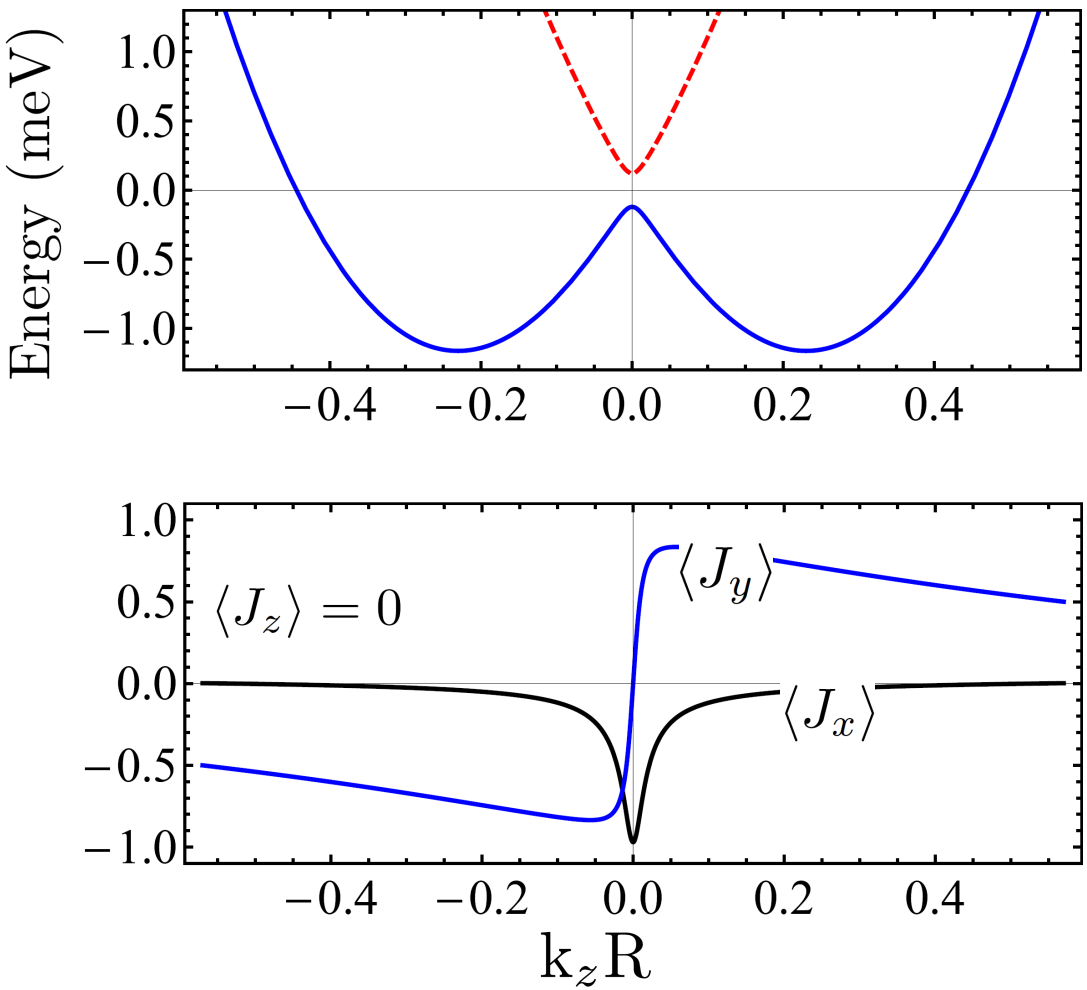
R_c = 5.0 nm
R_s = 6.5 nm

No RSOI!

Helical Hole States & Giant SOI

Kloeffel, Trif, and DL, Phys. Rev. B 84, 195314 (2011)

E-field along *x*: E_x = 6 V/μm ➔ strong Rashba SOI (~ 1-10 meV)



B_x opens a gap

0.8 T: ~ 0.25 meV
0.3 T: ~ 0.10 meV

R_c = 5.0 nm
R_s = 6.5 nm

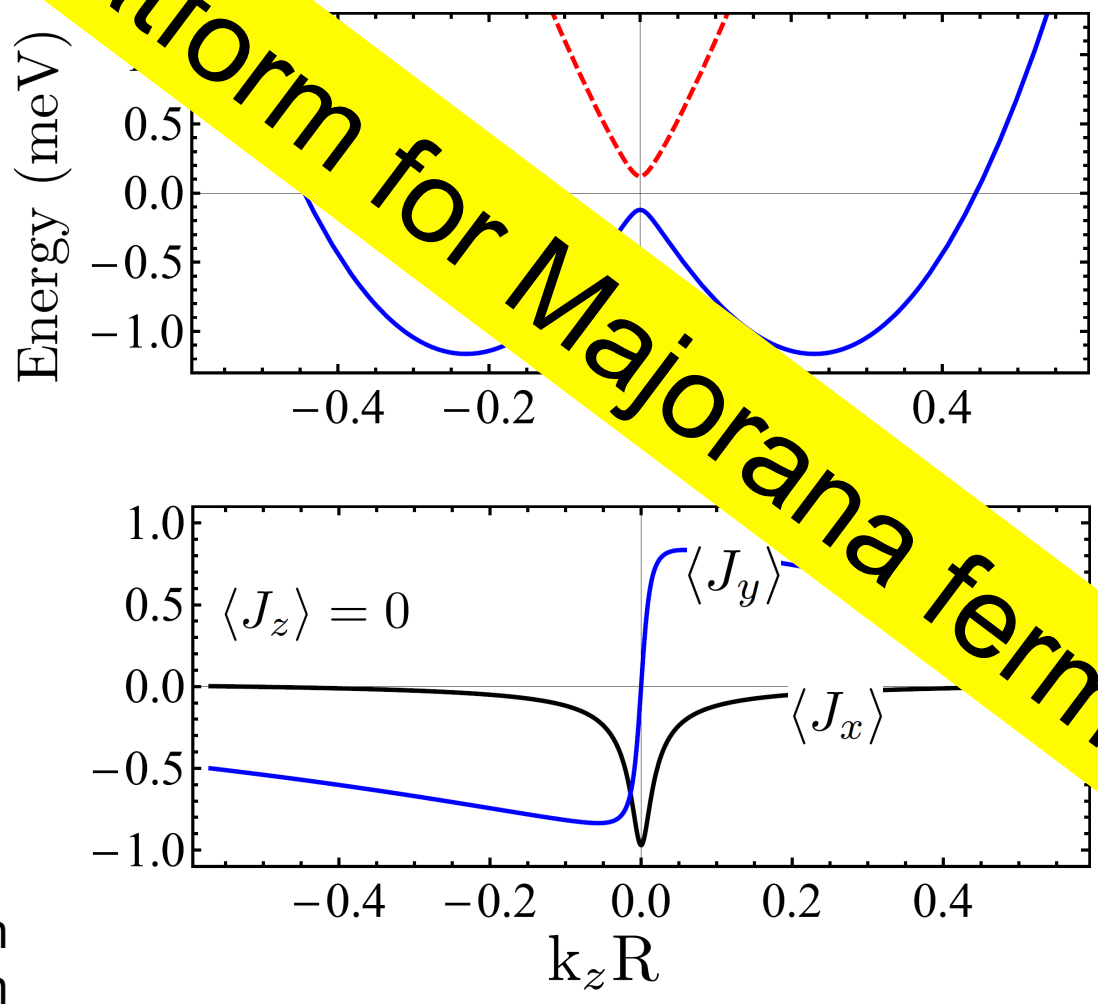
No RSOI!

Helical Hole States & Giant SOI

➔ platform for Majorana fermions!

loeffel, Trif, and DL, Phys. Rev. B 84, 195314 (2011)

E-fie… E_x = 6 V/μm ➔ strong Rashba SOI (~ 1-10 meV)



B_x opens a gap

0.8 T: ~ 0.25 meV
0.3 T: ~ 0.10 meV

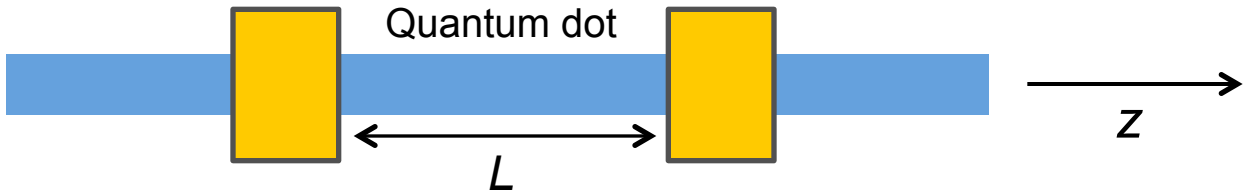
R_c = 5.0 nm
R_s = 6.5 nm

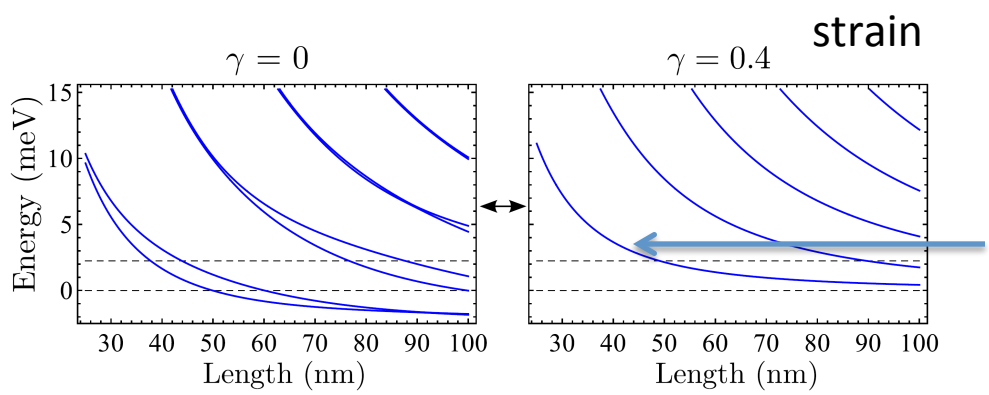
No RSOI!

Quantum Dots in SiGe nanowire

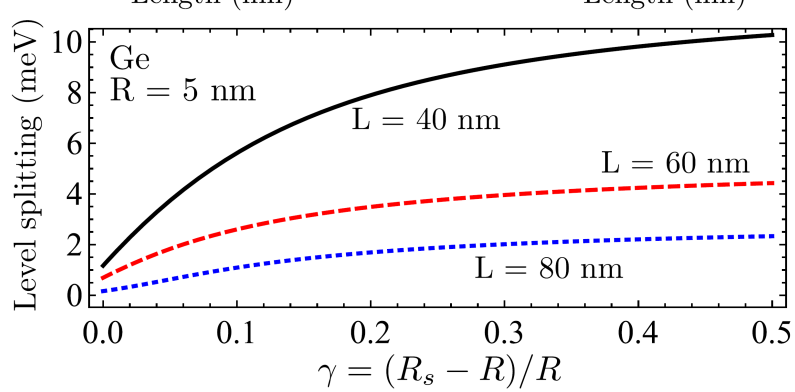
Kloeffel, Trif, and DL, Phys. Rev. B 84, 195314 (2011)

Add confinement along the *z* direction



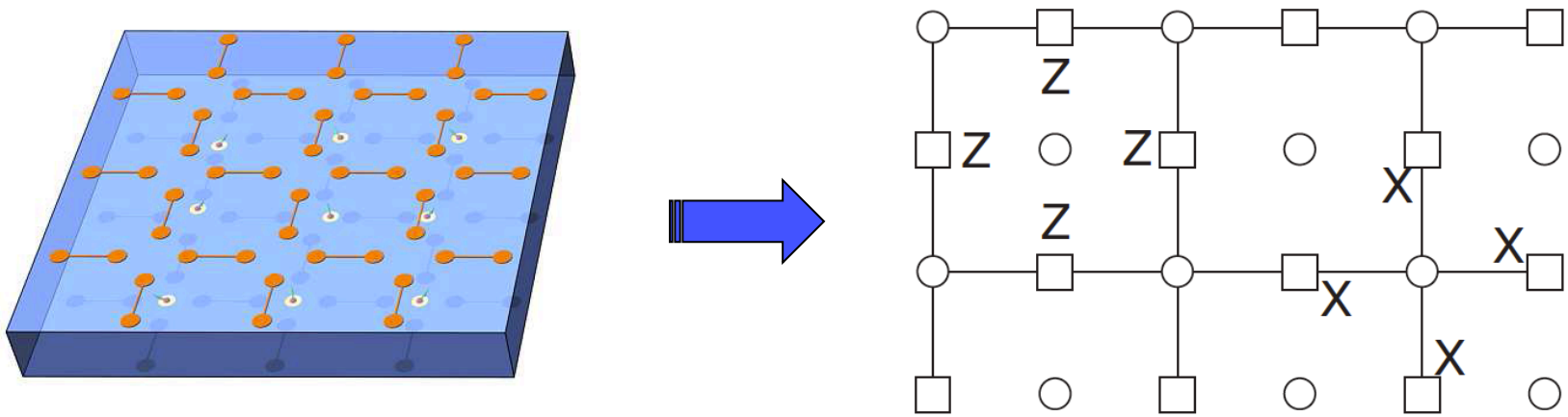


each line 2-fold degenerate ➔ spin qubit



2D architecture ➔ surface code

Trifunovic et al., arxiv: 1110.1342

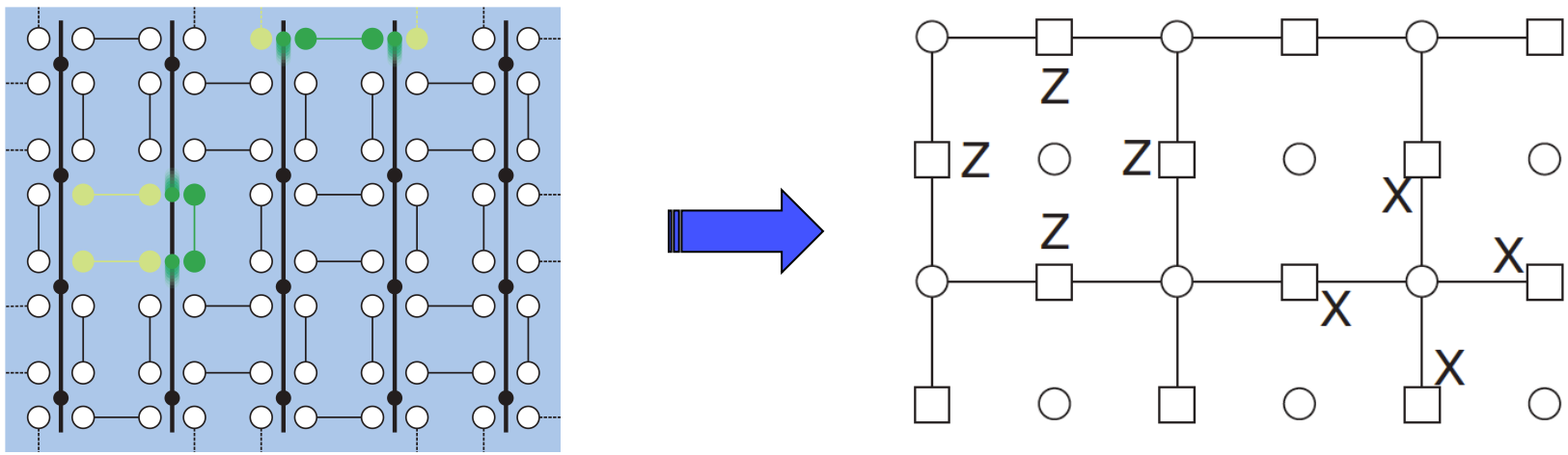


Surface code has error threshold of 1.1 % !

Raussendorf and Harrington, PRL 98,190504 (2007)
Wang, Fowler, and Hollenberg, PRA 83, 020302 (2011)

2D architecture ➔ surface code

Trifunovic et al., arxiv: 1110.1342



Surface code has error threshold of 1.1 % !

Raussendorf and Harrington, PRL 98,190504 (2007)
Wang, Fowler, and Hollenberg, PRA 83, 020302 (2011)

Hybrid system: LD-ST coupling

Trifunovic, Day, Trif, Wootton, Abebe, Yacoby, DL, arxiv: 1110.1342 (PRX, in print)

Bringing the best of the two worlds together....

$$H_{hybrid} = \frac{3\mathrm{m}\mu_B g J(\gamma \times B) \cdot \sigma}{4\omega_x^2}\tau_z$$

➔ construct SWAP gate between LD spin-qubit and ST qubit

Outline

- spin qubits and quantum dots: GaAs and others

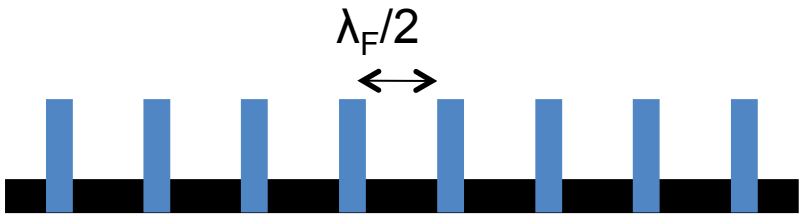
- long-distance spin-spin coupling (floating gates)

  ➔ **scalable 2D architecture**

- (Exotic) Bound states in CDW-wires as quantum dots

Novel ‘Quantum Dots’: CDW bound states

Suhas Gangadharaiah, Luka Trifunovic, and DL, arXiv:1111.5273



Tamm-Shockley bound states at wire ends (1932/39)

FIG. 1. The figure shows a quantum wire (black) of length L with negatively charged gates (blue) forming a superlattice potential. Due to the induced charge density modulation a bound state at each wire end can emerge.

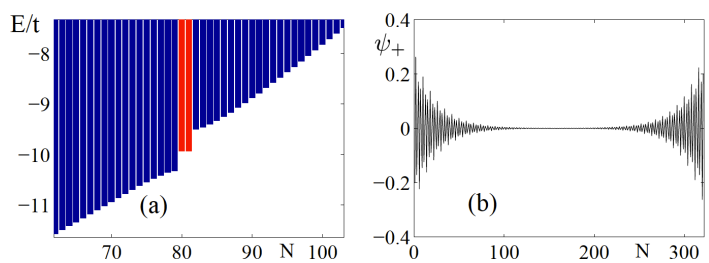


FIG. 2. (a) The part of the spectra around the gap of the Hamiltonian given by Eq. (1), obtained by exact diagonalization. The red bars denote two almost degenerate bound (midgap) states. We have chosen for the parameters $t = 7$, $\Delta = 0.8$, and $N = 320$. (b) One of two bound states. Plotted here is $\psi_+ = \psi_L + \psi_R$, where $\psi_{L,R}$ are states localized at the left (right) end of the wire.

CDW bound states: Jackiw-Rebbi Fermions

Suhas Gangadharaiah, Luka Trifunovic, and DL, arXiv:1111.5273

linearize: $\Psi_\sigma(x) = \mathcal{R}_\sigma(x)e^{ik_F x} + \mathcal{L}_\sigma(x)e^{-ik_F x}$

finite wire: $\mathcal{R}_\sigma(x) = -\mathcal{L}_\sigma(-x).$

kinetic term: $H_0^{(1)} = -iv_F \int_{-L}^{L} dx \mathcal{R}_\sigma^\dagger(x)\partial_x \mathcal{R}_\sigma(x)$

CDW term: $H_0^{(2)} = \Delta_0 \int_0^L dx \cos(2k_{CDW}x + \vartheta)\Psi_\sigma^\dagger(x)\Psi_\sigma(x)$

compact: $H_0 = (1/2)\int_{-L}^{L} dx \mathbf{R}_\sigma^\dagger \mathcal{H}_0 \mathbf{R}_\sigma$ $\quad \mathbf{R}_\sigma(x) = [\mathcal{R}_\sigma(x), \mathcal{R}_\sigma(-x)]^T$

Jackiw-Rebbi Hamiltonian

PRD (1976)

$$\mathcal{H}_0 = -iv_F\tau_z\partial_x + m_1(x)\tau_x + m_2(x)\tau_y$$

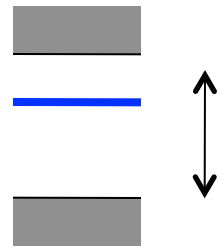
$$m_1(x) = -\cos[2\delta kx + \vartheta \mathrm{sgn}(x)]\Delta_0/2,$$
$$m_2(x) = \sin[2\delta kx + \vartheta \mathrm{sgn}(x)]\Delta_0/2,$$

[JR bound states: fermions with fractional charge e/2]

Bound state is spinful fermion

$$\psi_\sigma \sim \exp[-i(\Delta_0 \exp[-i\vartheta]/2v_F)x]$$

$$\boxed{\varepsilon = -\frac{\Delta_0}{2}\cos\vartheta}$$



Δ_0 : CDW gap

1. Bound state is stable against fluctuations of CDW amplitude $\delta(x)$ and phase $\theta(x)$:

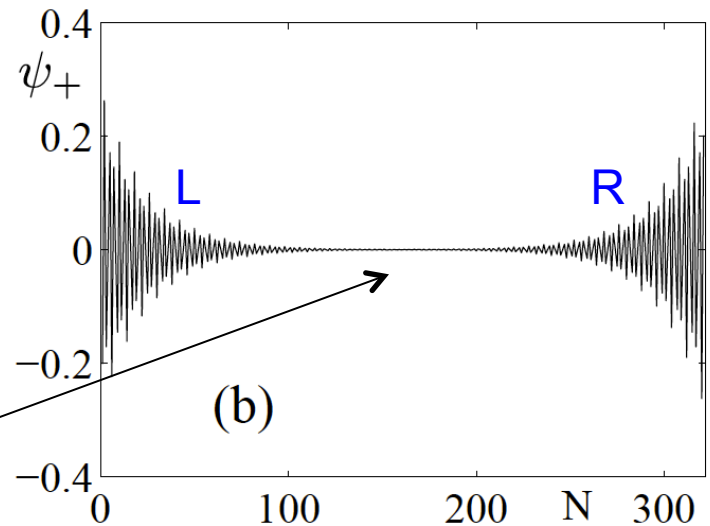
$$\delta\epsilon = -\frac{\Delta_0}{4v_F}\int_0^\infty dx\delta(x)\sin 2\vartheta(x)e^{-\Delta_0 \sin[\vartheta_0]x/v_F} \ll \varepsilon$$

2. e-e interactions *decrease* localization length (use LL theory):

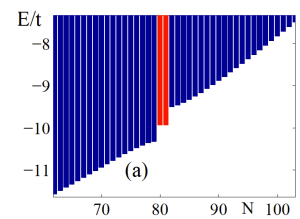
$$\xi \sim a(a\Delta_0/v_F)^{2/(K_c-3)}$$

Two bound end states form effective double dot system:

Gangadharaiah, Trifunovic, and DL, arXiv:1111.5273



Note: only one single orbital state per 'dot'



overlap t=t(Δ)

$$H = -t\sum_{\sigma=\uparrow,\downarrow}(c^{\dagger}_{\sigma,R}c_{\sigma,L} + h.c.) + U\sum_{i=L,R} n_{\uparrow,i}n_{\downarrow,i},$$

Δ >> U >> t : $\boxed{H \approx J\vec{S}_L \cdot \vec{S}_R}$ $J(\Delta) = 4t^2/U$

➔ CNOT gate

Conclusions

- spin qubits and quantum dots: GaAs and others

- long-distance spin-spin coupling (floating gates)

  ➔ **scalable 2D architecture**

- (Exotic) Bound states in CDW-wires as quantum dots