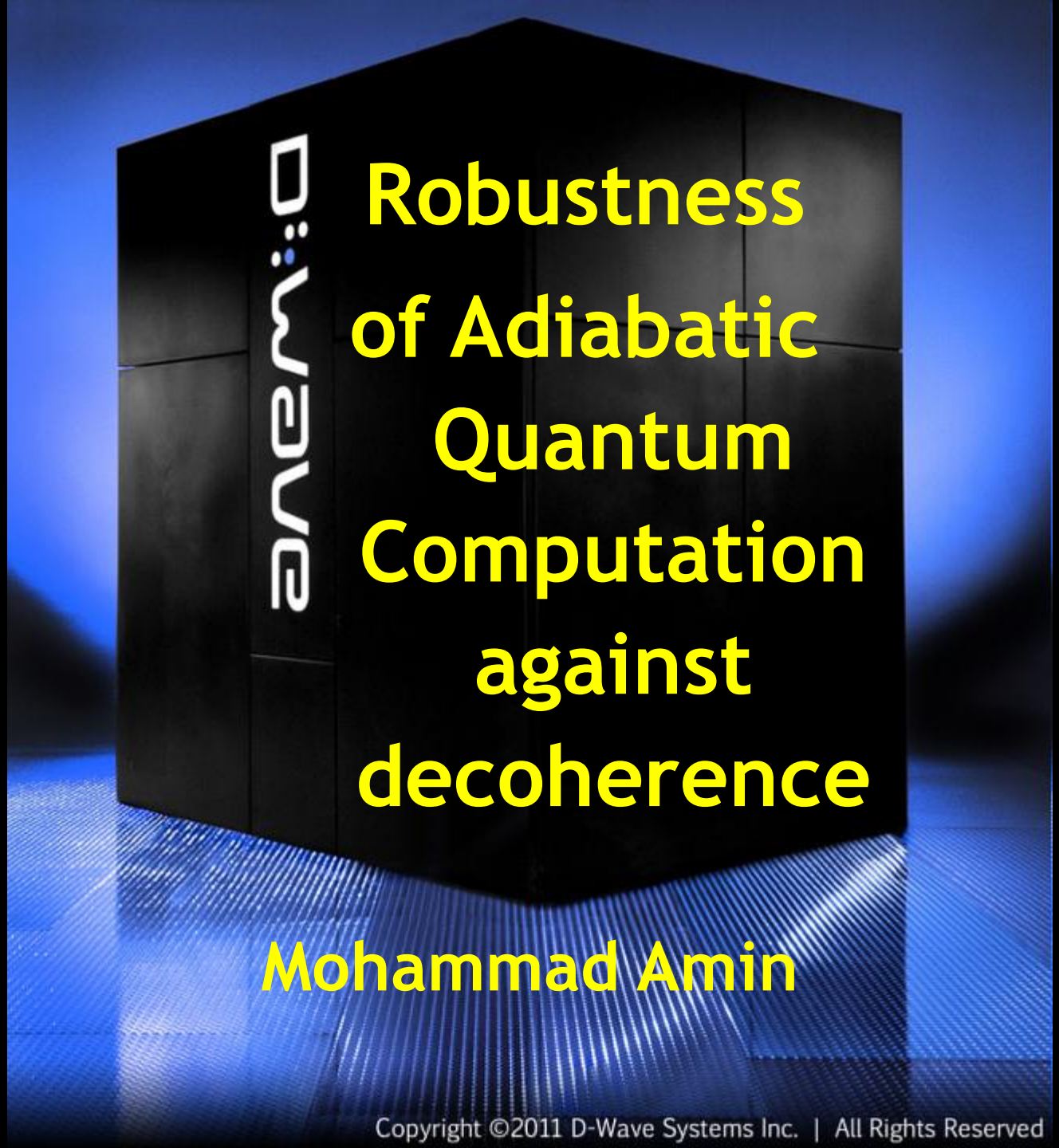
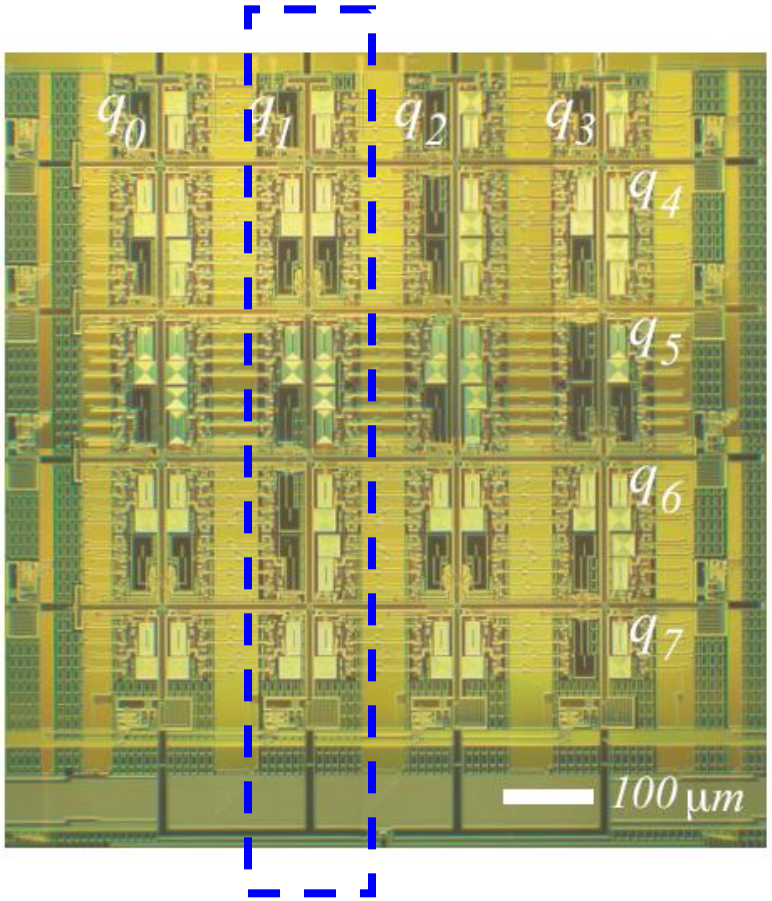
Robustness of Adiabatic Quantum Computation against decoherence

Mohammad Amin

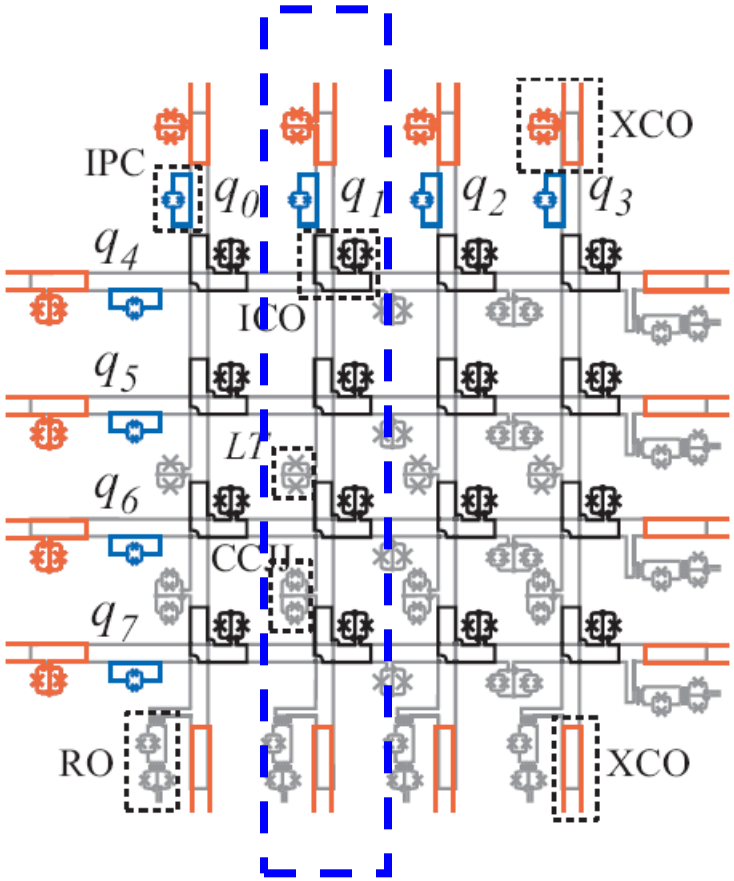


Eight Qubit Unit Cell

Qubit #1



Qubit #1



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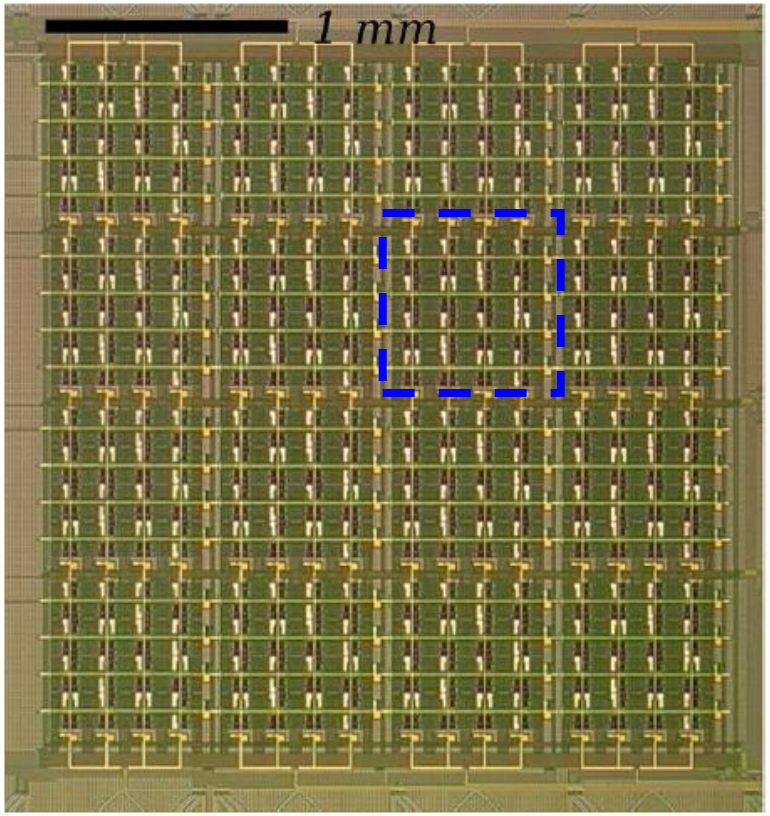
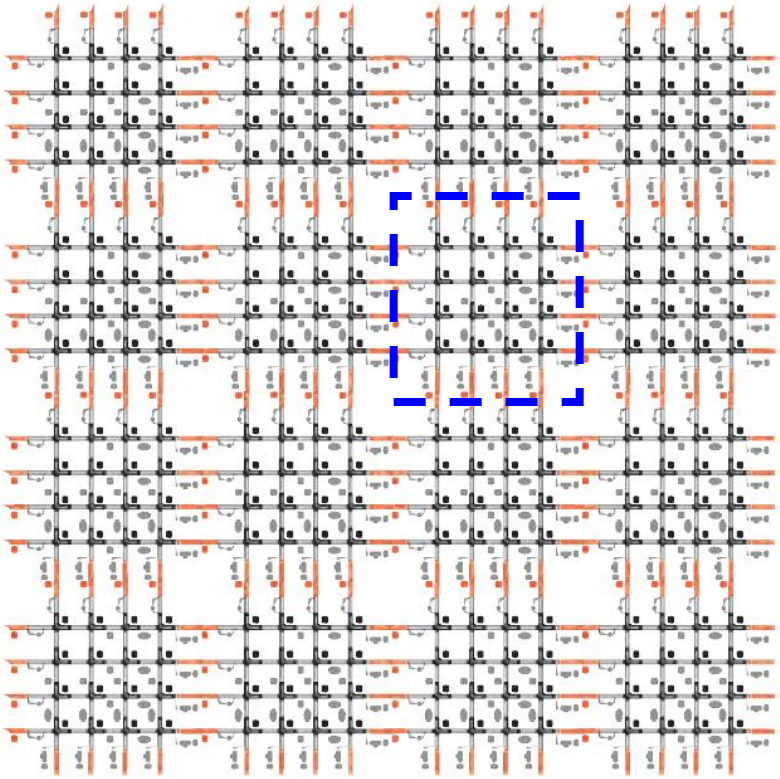
128-Qubit Chip

Unit cell



Unit cell



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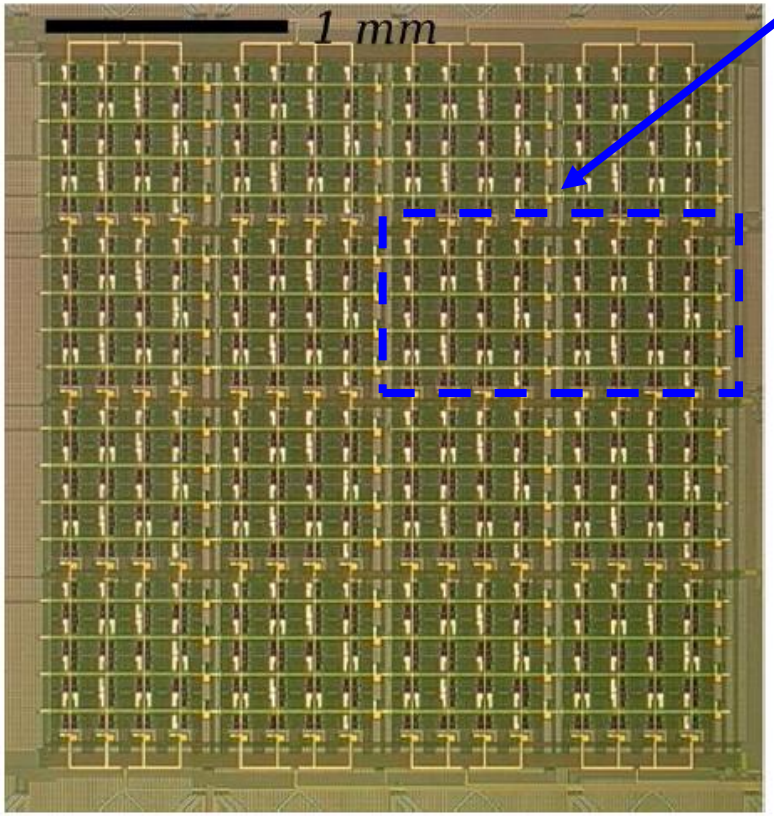
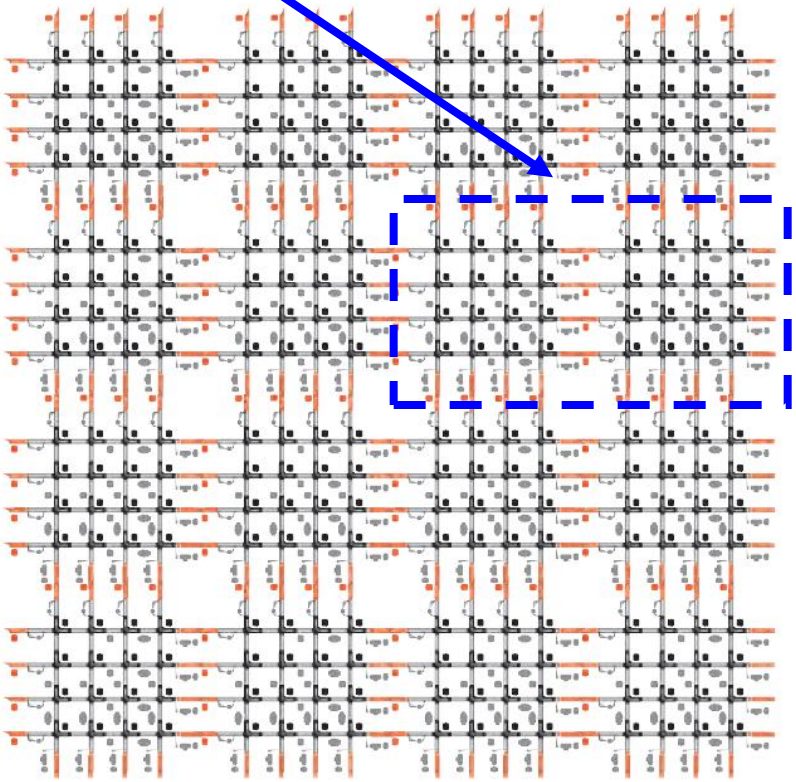
128-Qubit Chip

16 Qubits used in this experiment:





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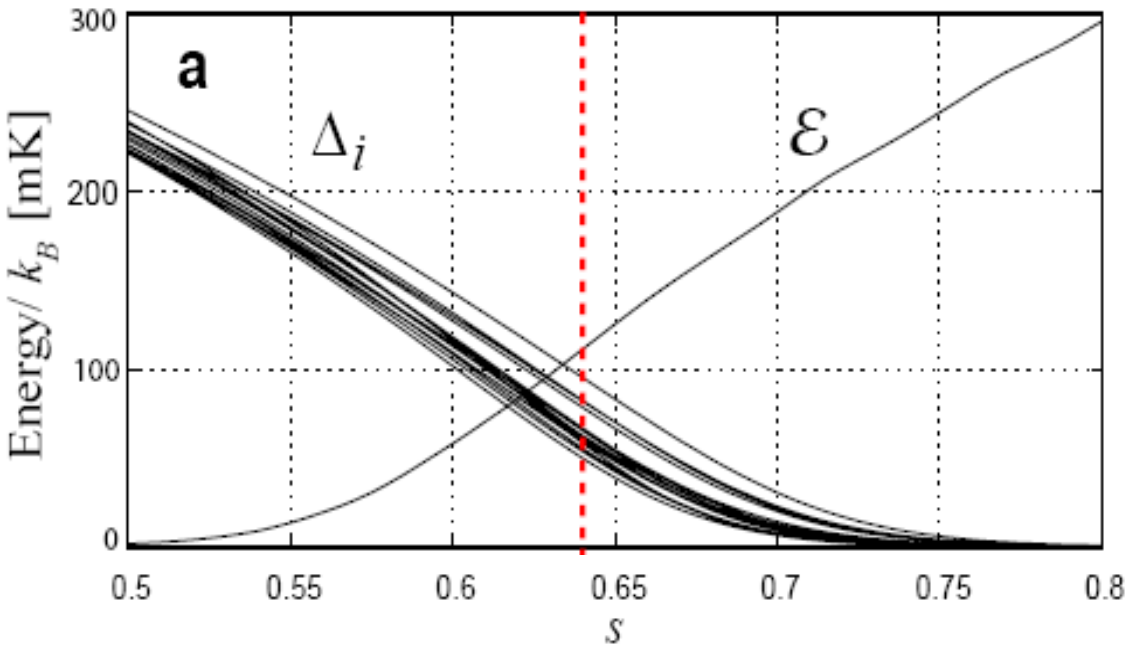
Hardware Hamiltonian

$$H = -\frac{1}{2}\sum_{i=1}^{N}\Delta_i(s)\sigma_i^x + \frac{1}{2}\mathcal{E}(s)H_P \qquad s = t/t_f$$

Ising Hamiltonian:

$$H_P = \sum_{i=1}^{N} h_i\sigma_i^z + \sum_{i,j=1}^{N} J_{ij}\sigma_i^z\sigma_j^z$$



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Published Information:

Qubit:

Harris et al., PRB 81, 134510 (2010)

Coupler:

Harris et al., PRL 98, 177001 (2007)

On-chip superconducting digital control:

Johnson et al., Supercond. Sci. Technol. 23, 065004 (2010)

Quantum annealing with 8-qubit unit cell:

Harris et al., PRB 82, 024511 (2010)

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IS IT QUANTUM?

Single-qubit measurements:

Harris et al., PRL 101, 117003 (2008)
Johansson et al. PRB 80, 012507 (2009)
Harris et al., PRB 81, 134510 (2010)
Lanting et al., PRB 83, 180502(R) (2011)

Two-qubit measurement:

Lanting et al., PRB 83, 180502(R) (2011)

Eight-qubit measurements:

Harris et al., PRB 82, 024511 (2010)
Johnson et al., Nature 473, 194 (2011)

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Outline

- **Open quantum systems**
- **Effect of environment on AQC**
- **16-qubit experimental results**

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S. Uchaikin
J. Wang
B. Wilson
G. Rose

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Open Quantum System

Hamiltonian: $$H(t) = H_S(t) + H_B + H_{SB}$$

System **Environment** **Interaction**

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Open Quantum System

Hamiltonian:

System **Environment** **Interaction**

In most QC schemes,
H_S is not the dominant term in H

Measurement based model: $H_S = 0$

Gate model: H_S **acts only on 1-2 qubits at a time**

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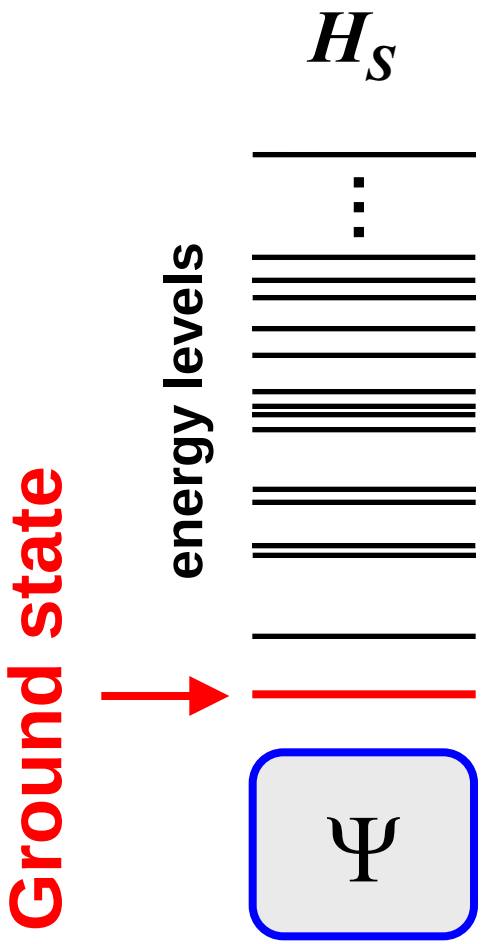
Using Ground State



1. H_S dominates H, therefore dominates the evolution

Eigenstates do not change by the Hamiltonian evolution

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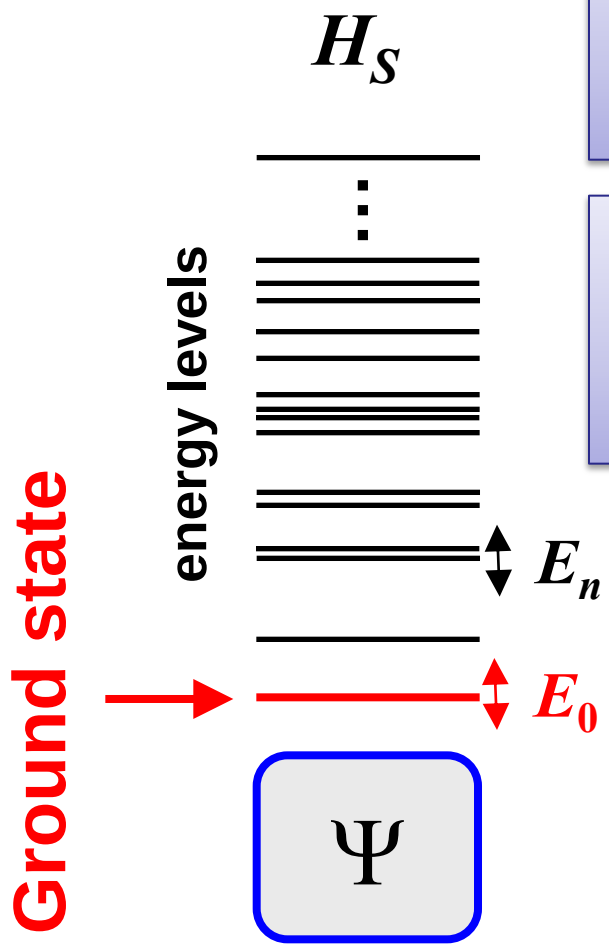
Using Ground State



1. H_S dominates H, therefore dominates the evolution

2. Such dephasing does not affect the wave function

Environment can cause energy fluctuations and therefore dephasing between energy levels

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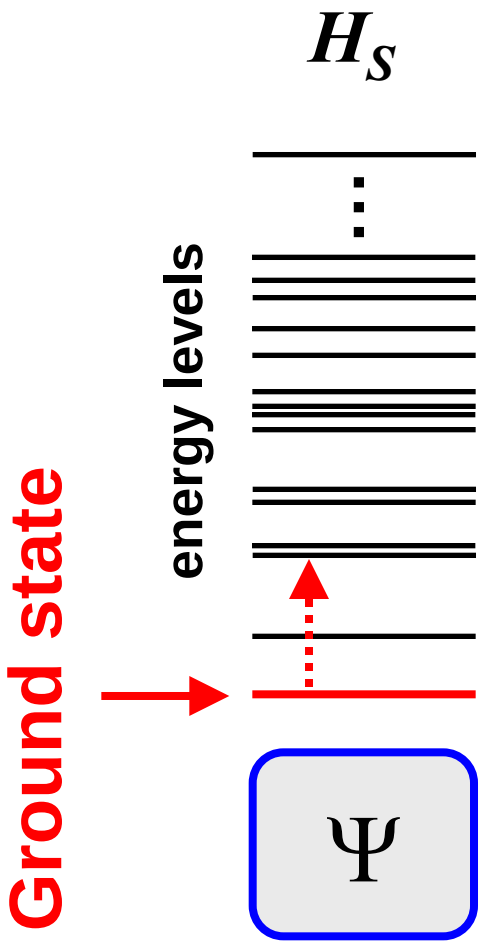
Using Ground State



1. H_S dominates H, therefore dominates the evolution

2. Such dephasing does not affect the wave function

Environment can cause excitation out of the ground state, i.e., error

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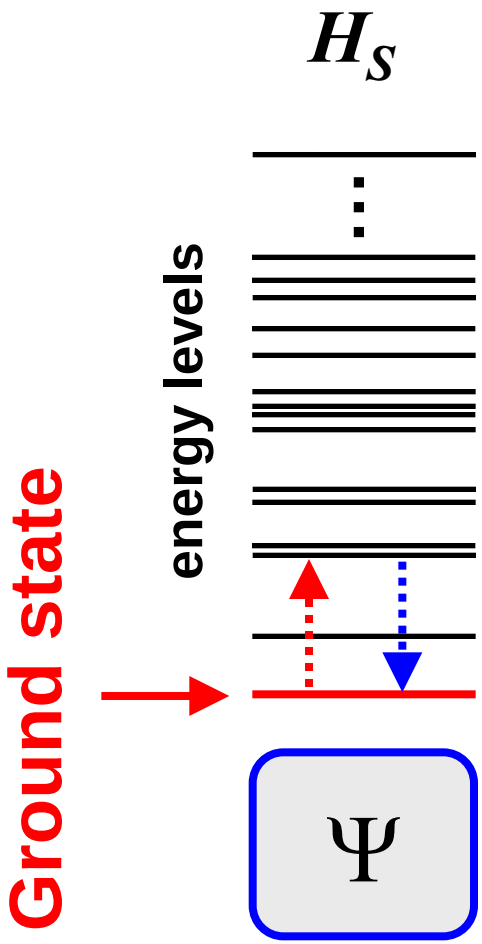
Using Ground State



1. H_S dominates H, therefore dominates the evolution

2. Such dephasing does not affect the wave function

3. Relaxation provides a natural self-error-correction

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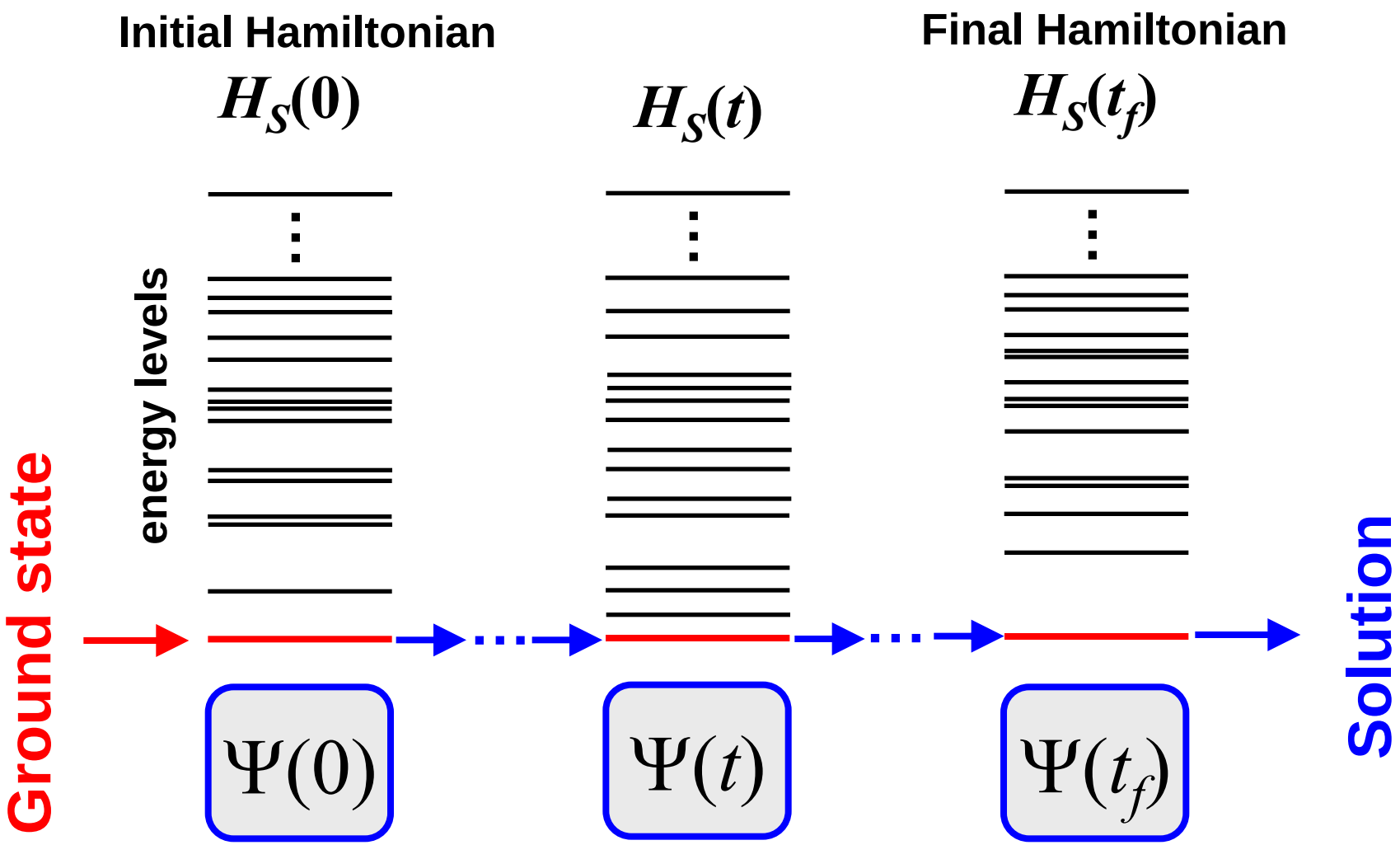
Ground State Computation



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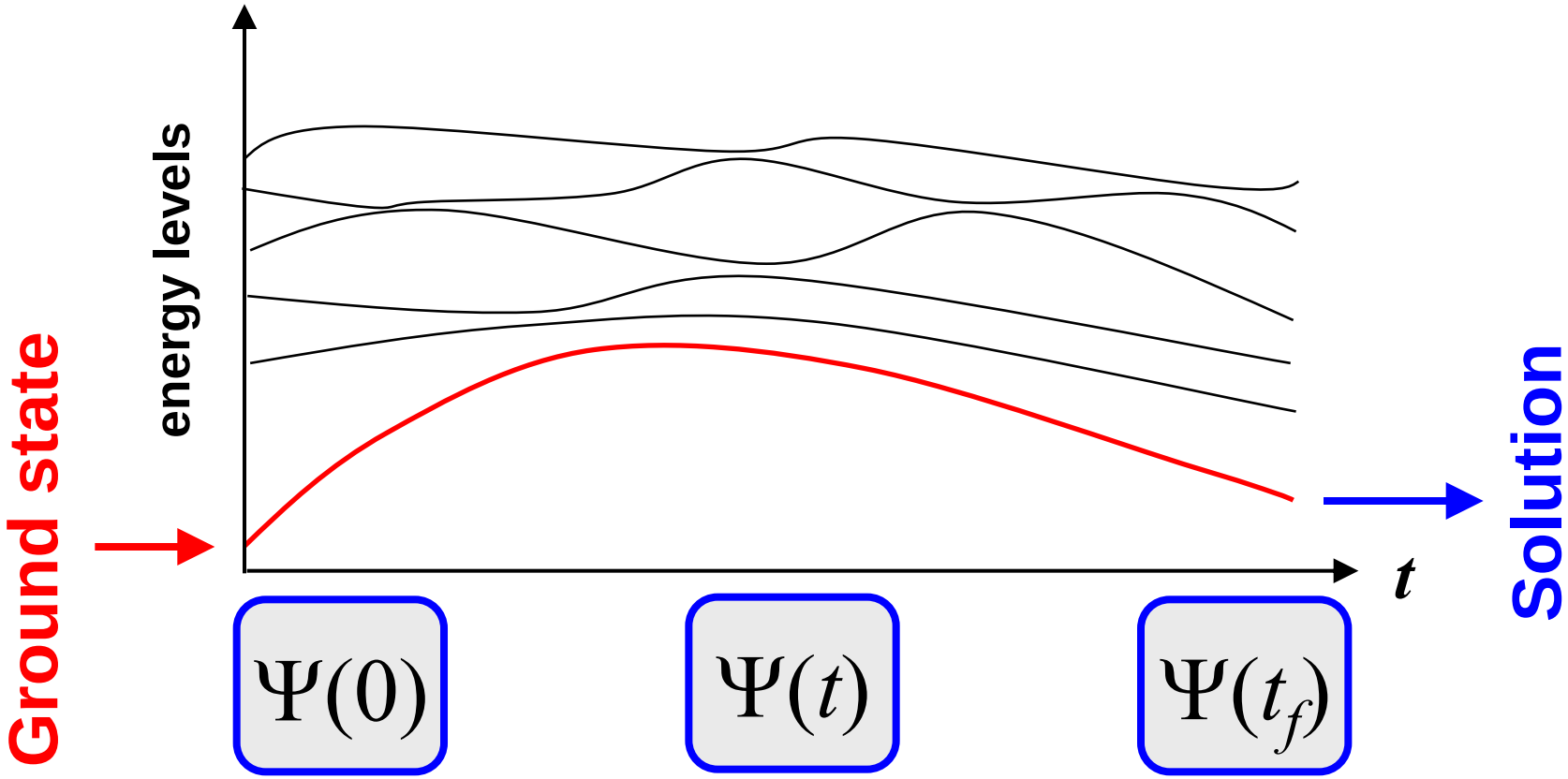
Adiabatic Quantum Computation

Farhi et al., Science 292, 472 (2001)

$$H_S(t) = (1-s)H_B + sH_P, \quad s = t/t_f$$



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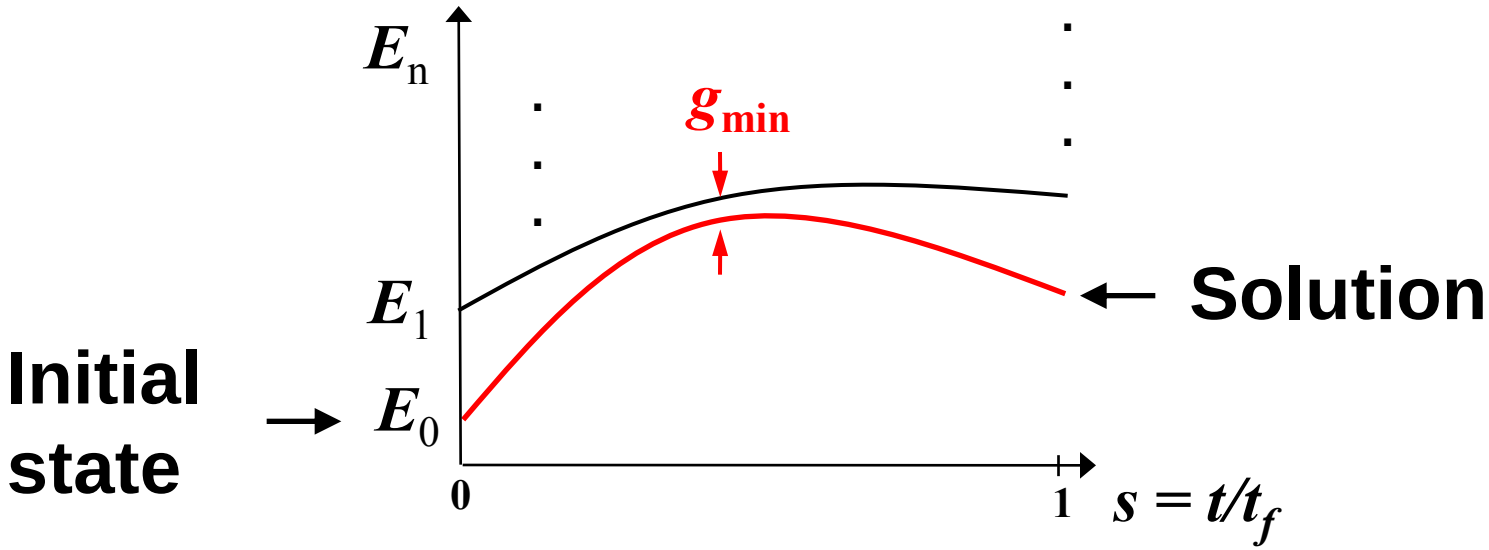
Adiabatic Theorem



Adiabatic Theorem:

System will stay in the ground state if

Evolution time: $$t_f >> \frac{\max_{s\in[0,1]} \left| \langle 0 | dH/ds | 1 \rangle \right|}{g_{min}^2}$$

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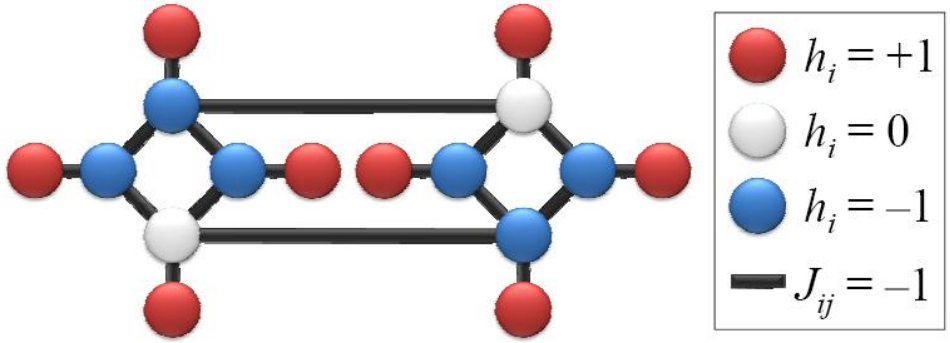
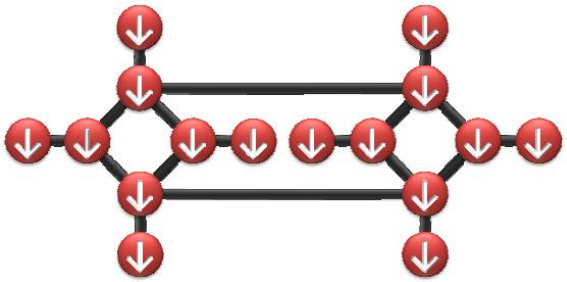
A 16-Qubit Small Gap Problem

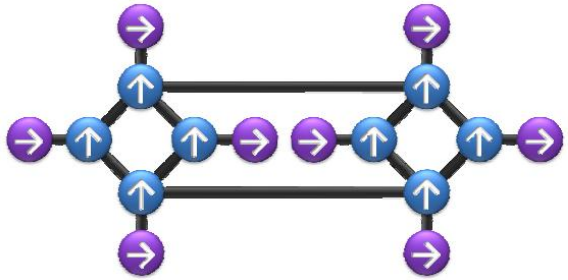


$\mathcal{E}$



$|\mathrm{GM}\rangle$

Final ground state
(global minimum of H_P)



$|\Sigma\rangle$

Final first excited state
(Superposition of 256 local minima)

$$|\rightarrow\rangle \equiv (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$$

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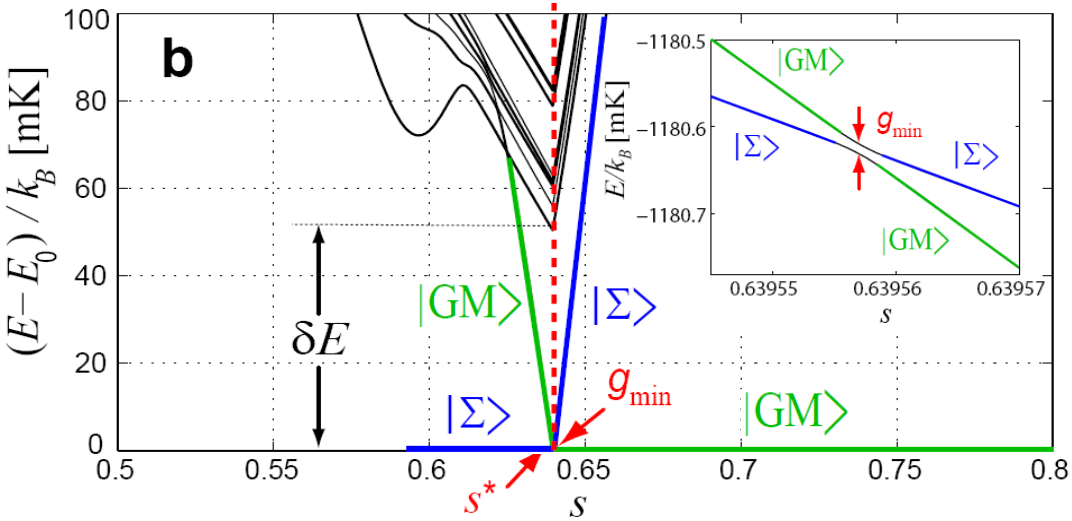
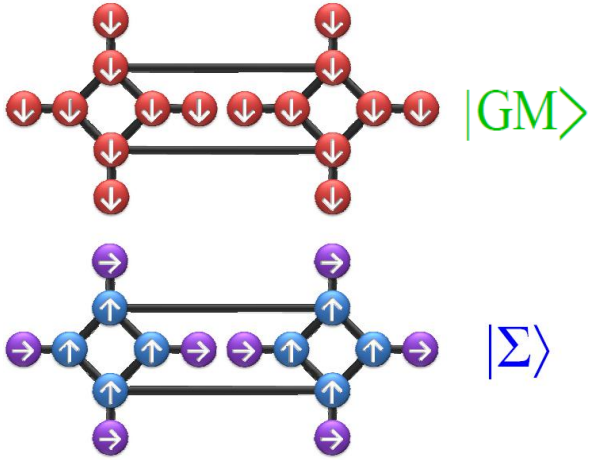
Energy Spectrum





$$g_{min} = 0.011 \text{ mK} << T$$

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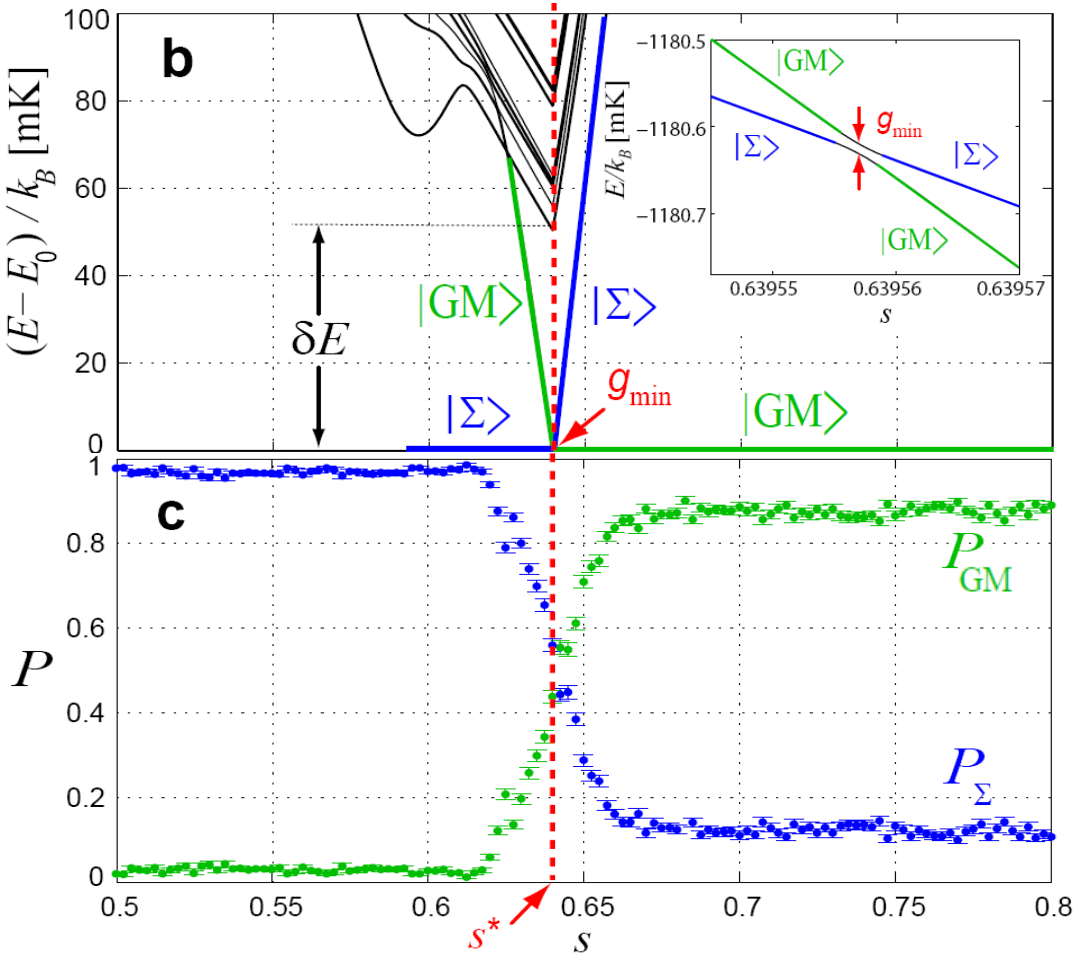
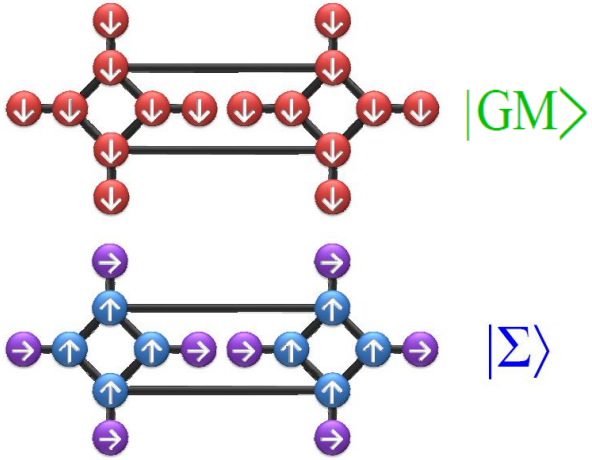
Crossing Probabilities





Probabilities cross at the anticrossing

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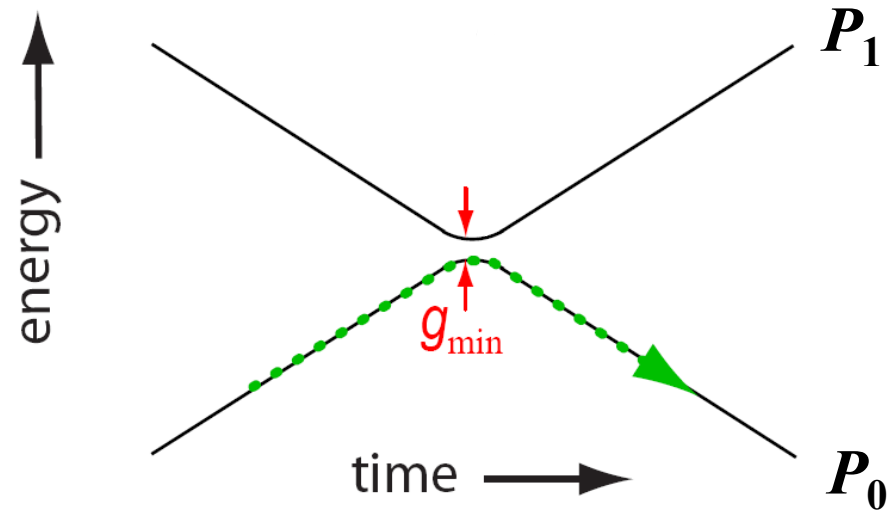
Two-State System

Hamiltonian:

$$H(t) \approx -\frac{1}{2}\left(vt\sigma_z + g_{\min}\sigma_x\right)$$

Pauli Matrices



Landau-Zener probability:

$$P_0 = 1 - e^{-\pi g_{\min}^2 / 2\hbar v}$$

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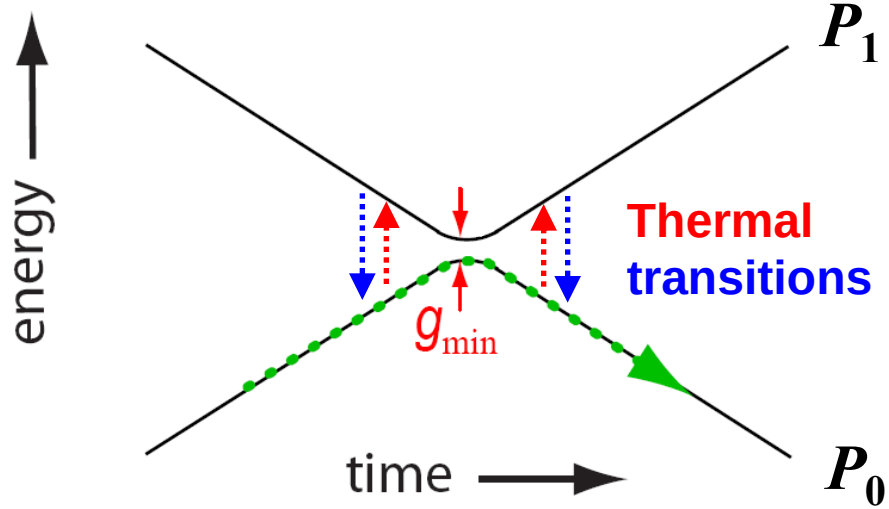
Open Quantum System (T>0)

Two state approximation:

$$H(t) \approx -\frac{1}{2}\left(\varepsilon\sigma_z + g_{\min}\sigma_x\right)$$



For $T >> g_{\min}$

At the anticrossing: $P_0 \sim 0.5$

What is the time scale to reach such a probability?

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Open Quantum System (T>0)

Computation time scale for open system
is of the same order
as that for a closed system

Ao and Rammer PRB 43, 5397 (1991)

Amin, Love, Truncik, PRL 100, 060503 (2008)

Amin, Truncik, Averin, PRA 80, 022303 (2009)

Amin, Averin, Nesteroff, PRA 79, 022107 (2009)

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Thermal Relaxation

Two state approximation: $$H(t) \approx -\frac{1}{2}\left(\varepsilon\sigma_z + g_{\min}\sigma_x\right)$$

Closed system:

$$P_0 = 1 - e^{-t_f/t_a}$$

Adiabatic time: $t_a \propto g_{\min}^{-2}$

Open system:

$$t_r^{-1} = \frac{g_{\min}^2}{\Omega^2} J(\Omega) \coth\frac{\Omega}{2T}$$

$$\Omega = \sqrt{\varepsilon^2 + g_{\min}^2}$$

Relaxation time: $t_r \propto g_{\min}^{-2}$

Similar time scales

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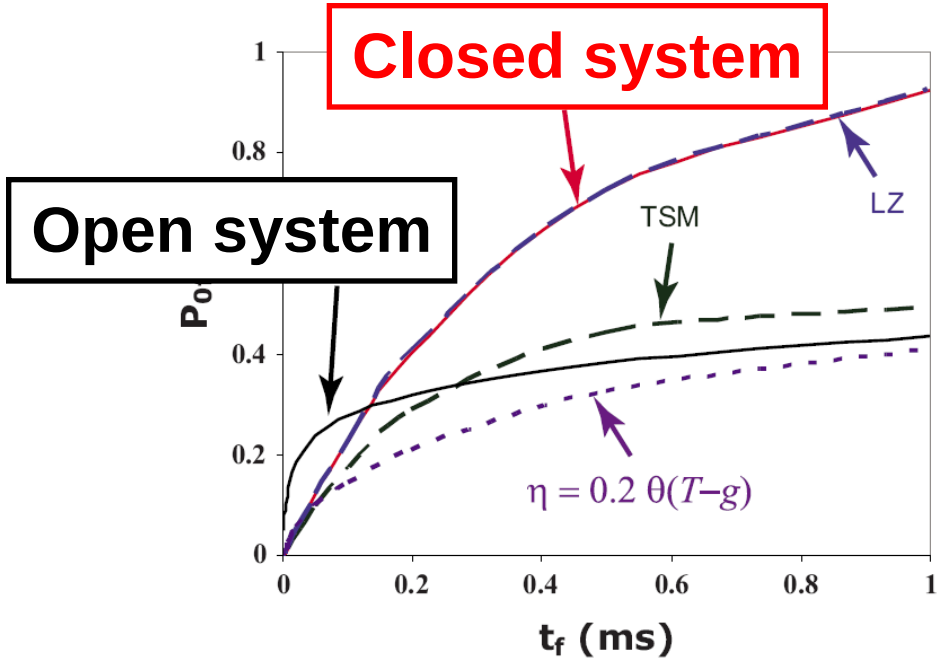
PHYSICAL REVIEW A **80**, 022303 (2009)

Role of single-qubit decoherence time in adiabatic quantum computation

M. H. S. Amin,[1] C. J. S. Truncik,[1] and D. V. Averin[2]



Similar porbabilities

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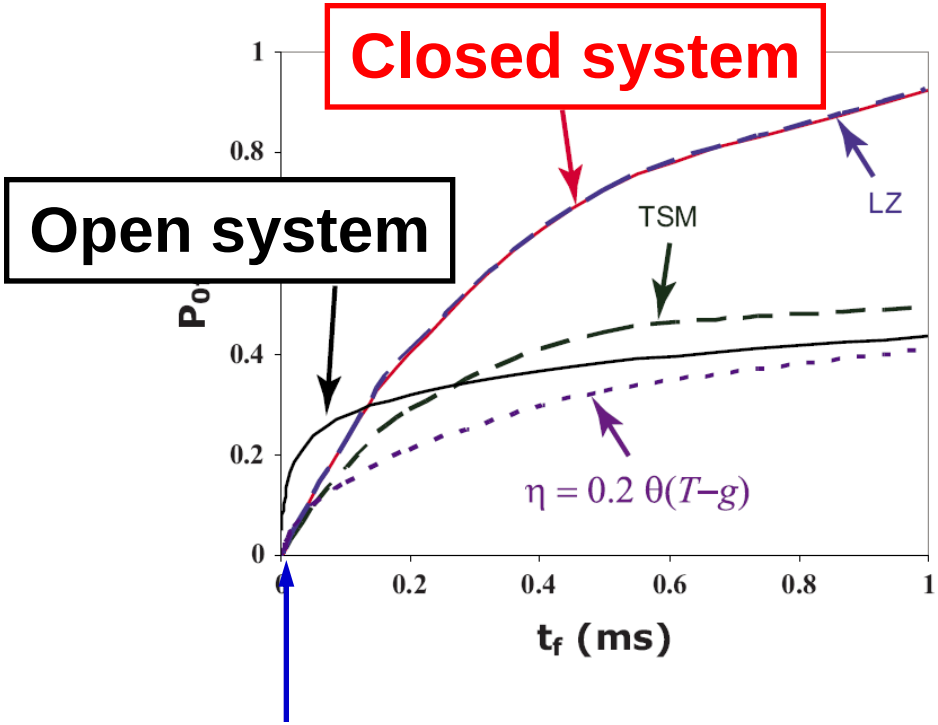
PHYSICAL REVIEW A **80**, 022303 (2009)

Role of single-qubit decoherence time in adiabatic quantum computation

M. H. S. Amin,[1] C. J. S. Truncik,[1] and D. V. Averin[2]



Decoherence time ~ 10 ns

$$t_f >> t_{decoherence}$$

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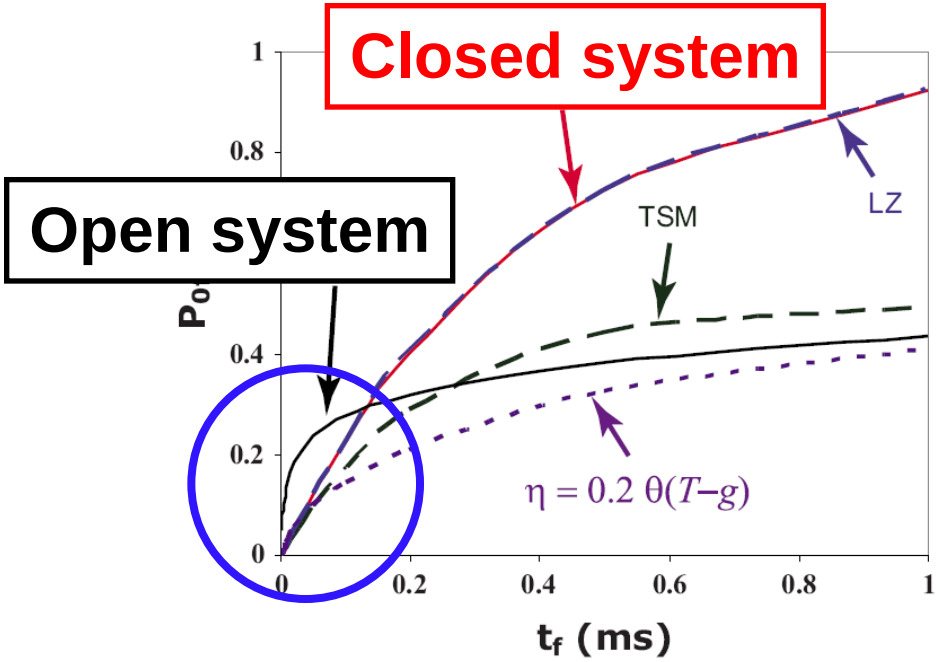
Environment May Help AQC!



PRL 100, 060503 (2008) PHYSICAL REVIEW LETTERS week ending 15 FEBRUARY 2008

Thermally Assisted Adiabatic Quantum Computation

M. H. S. Amin,[1,*] Peter J. Love,[1,2] and C. J. S. Truncik[1]

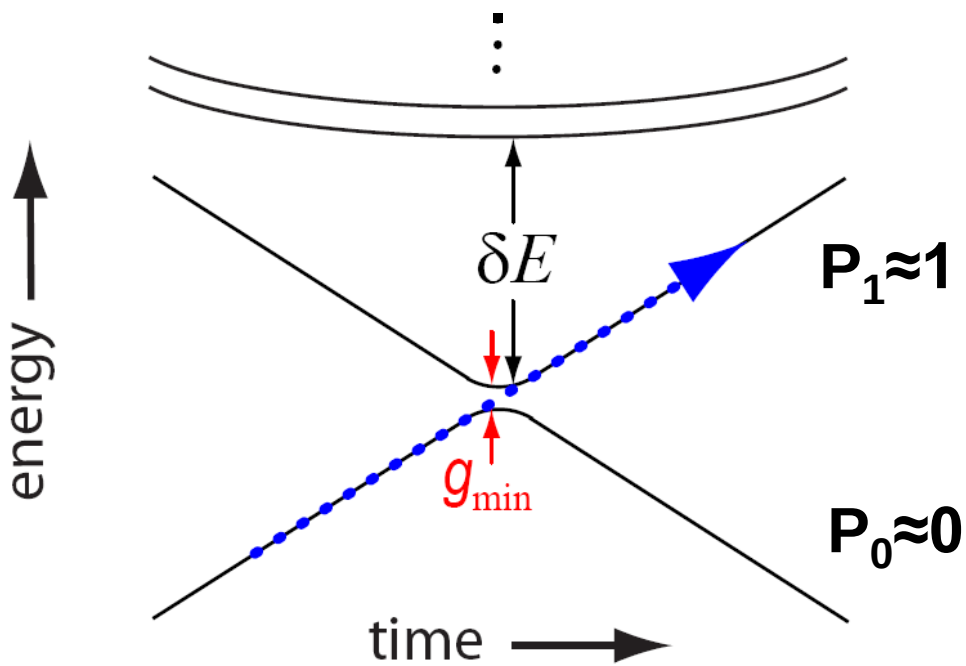
[1]*D-Wave Systems Inc., 100-4401 Still Creek Drive, Burnaby, British Columbia, V5C 6G9, Canada*

[2]*Department of Physics, Haverford College, 370 Lancaster Avenue, Haverford, Pennsylvania 19041-1392, USA*

(Received 30 April 2007; published 12 February 2008)

Closed System

Fast evolution:



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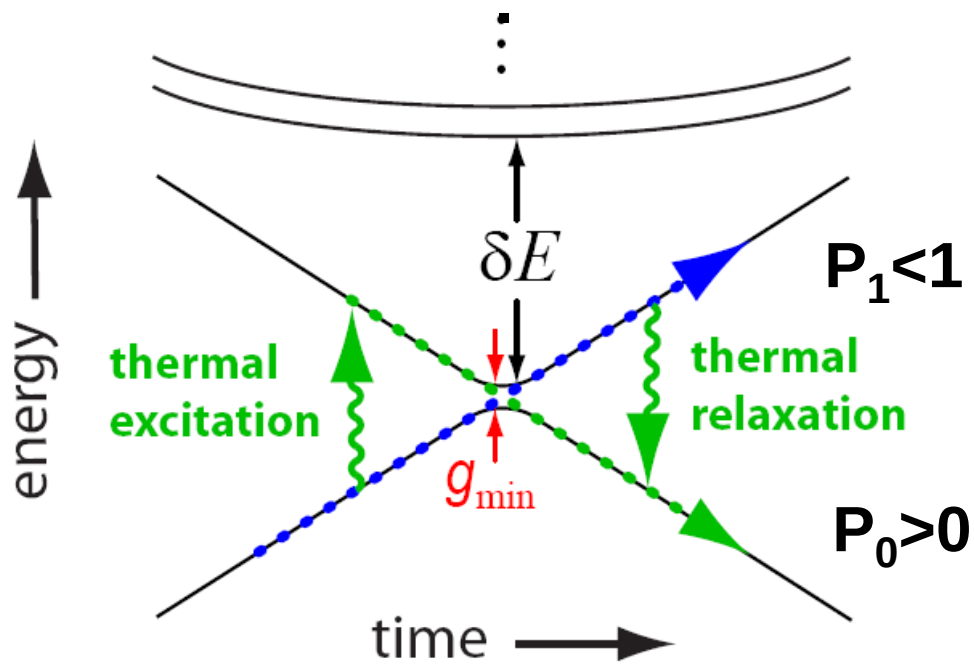
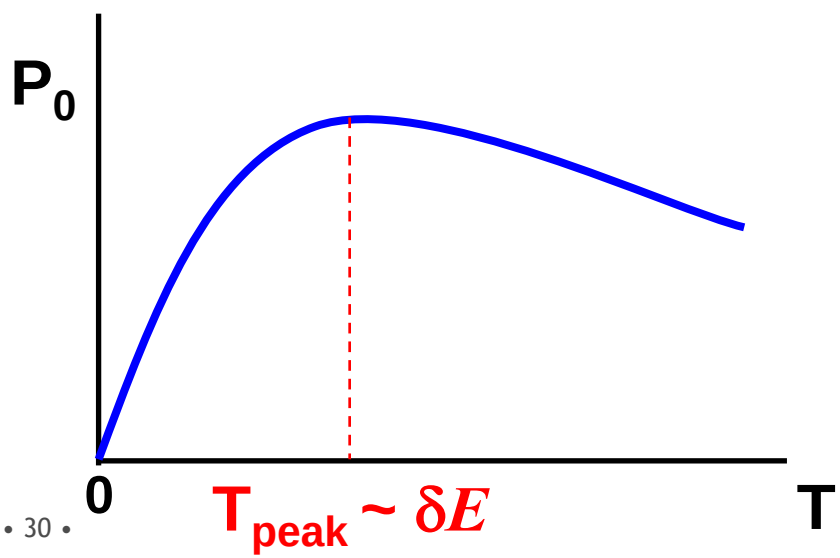
Environment at T>0

Fast evolution:





Raising T should increases the final ground state probability

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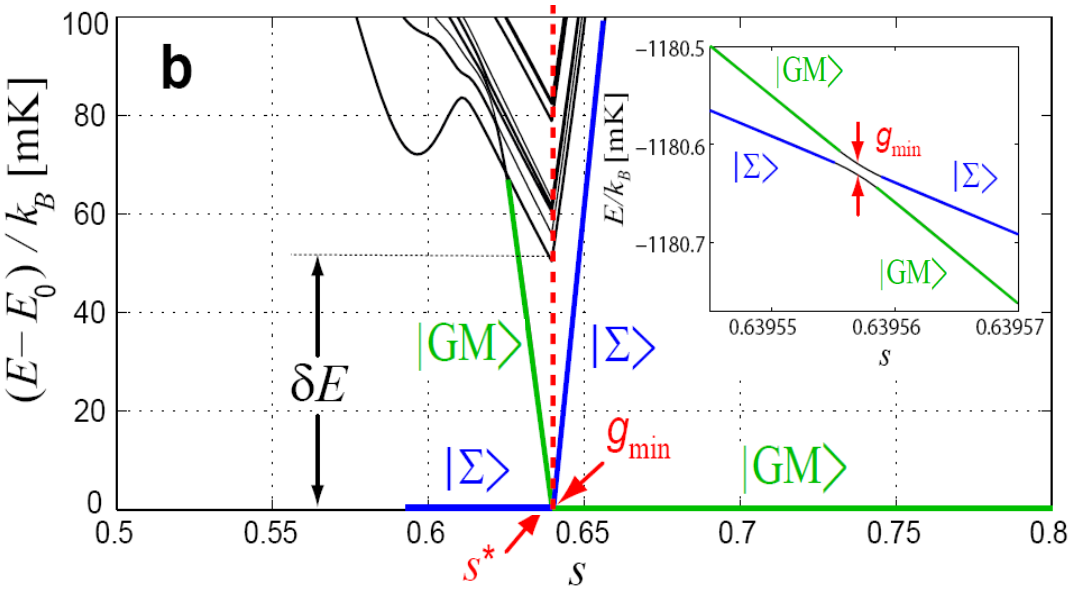
Thermally Assisted Performance



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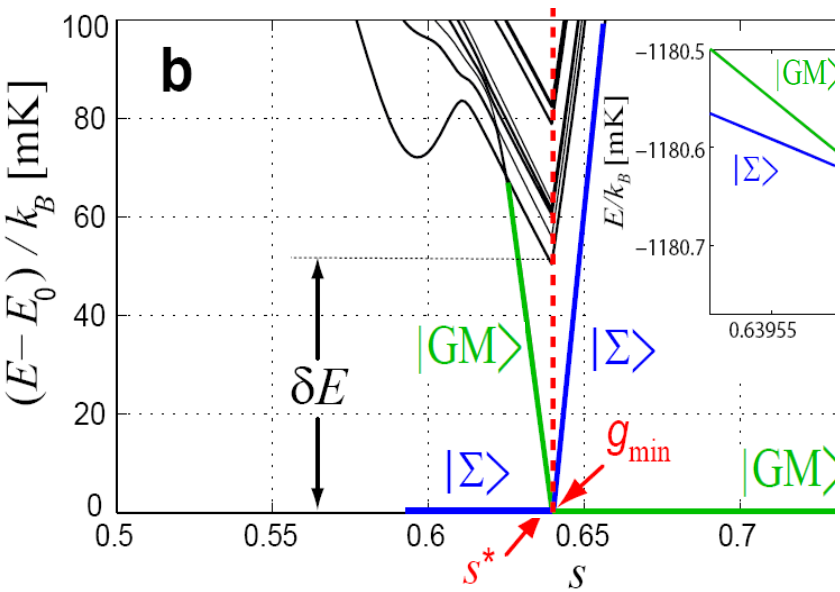
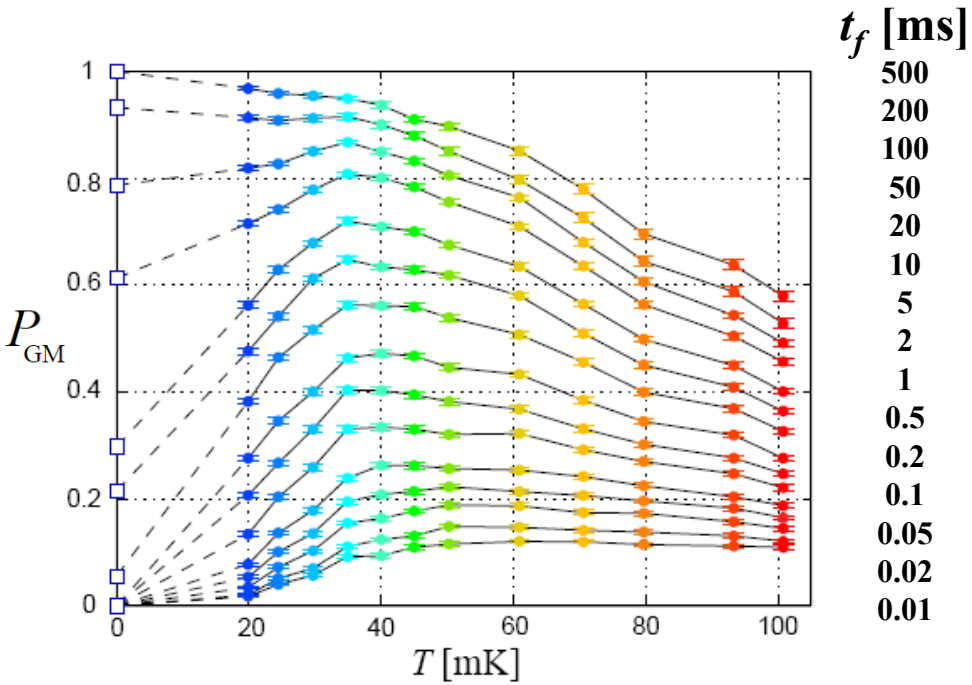
Thermally Assisted Performance





In agreement with theoretical predictions

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Landau-Zener Probability at T=0

$$H(t) \approx -\frac{1}{2}\left(vt\sigma_z + g_{\min}\sigma_x\right) - \frac{1}{2}Q\sigma_z$$

Coupling to environment

At T=0: open system probability is exactly the same as closed system LZ probability

Ao and Rammer PRB 43, 5397 (1991)

Wubs et al. PRL 97, 200404 (2006)

Wan, Amin, and Wang, Int. J. Quant. Inf. 7, 725 (2009)

Amin, Averin, Nesteroff, PRA 79, 022107 (2009)

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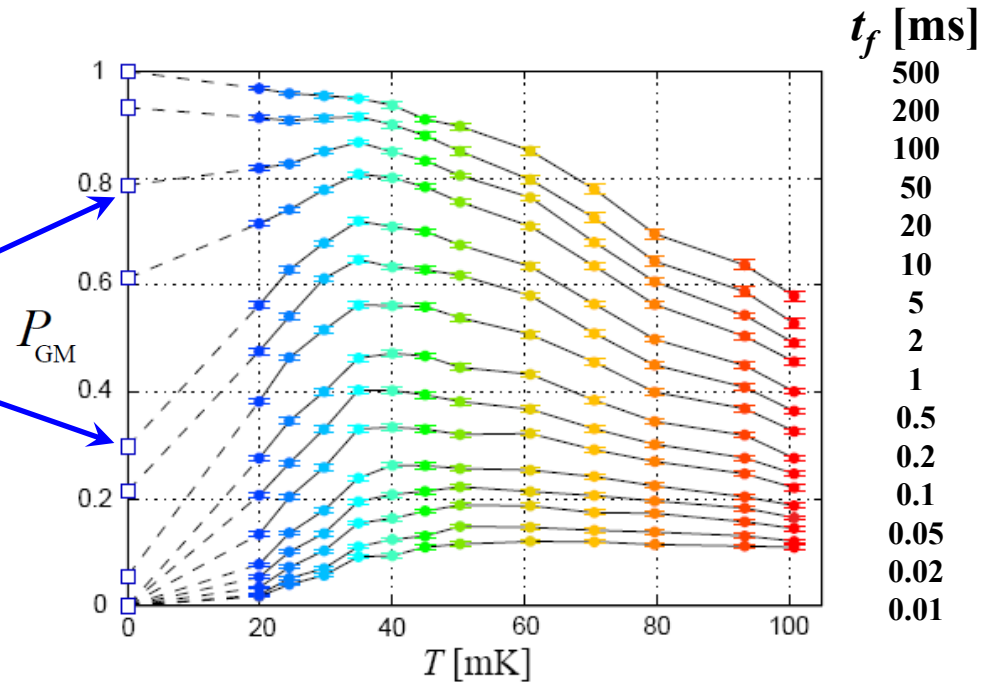
T=0 Behavior

Extrapolation to T=0



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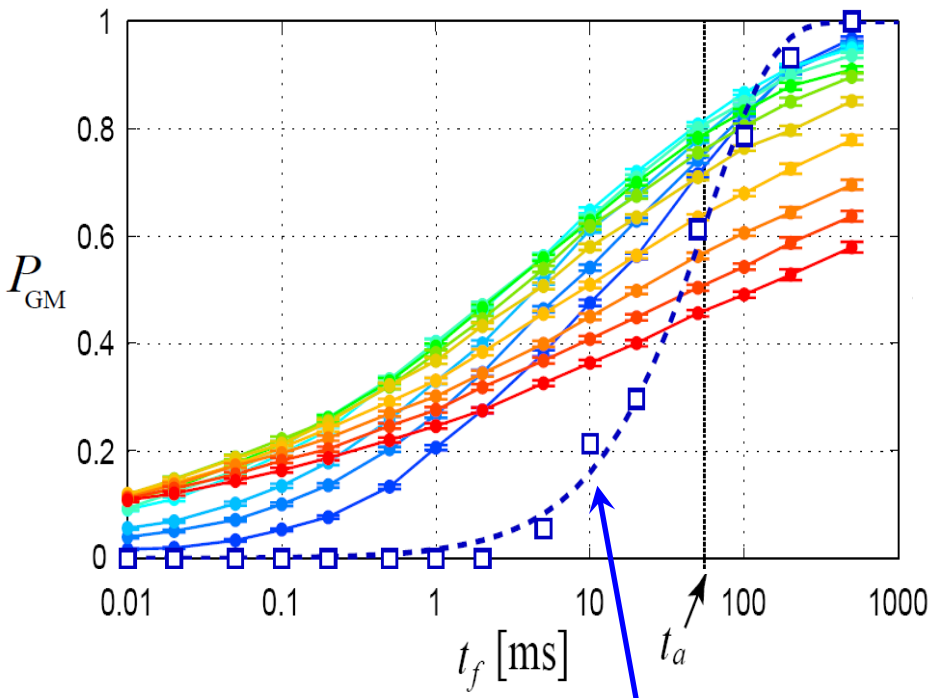
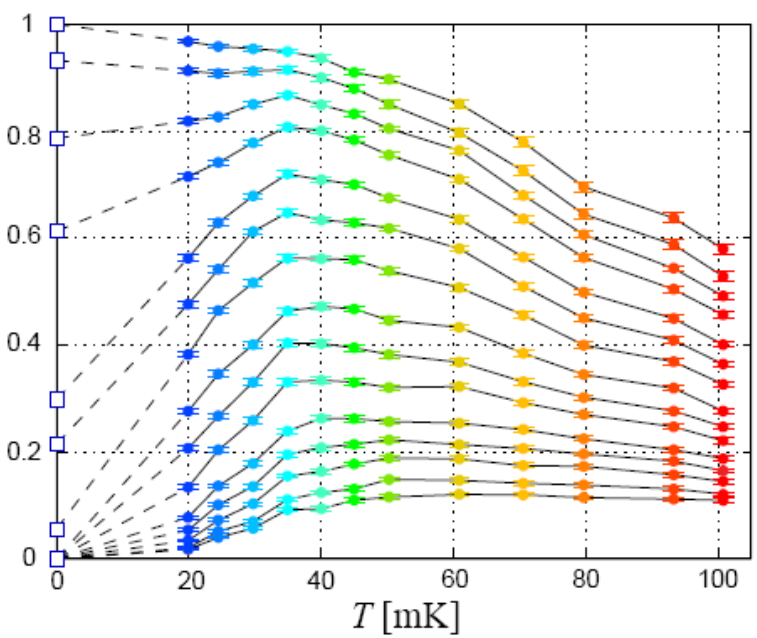
T=0 Behavior





t_f [ms]
500
200
100
50
20
10
5
2
1
0.5
0.2
0.1
0.05
0.02
0.01

Expected closed system curve

$$P_M \approx 1 - e^{-t_f/t_a}$$

$$t_a = 2\hbar\nu/\pi g_{\min}^2$$

$$\nu = |d(E_1 - E_0)/ds|_{s*}$$

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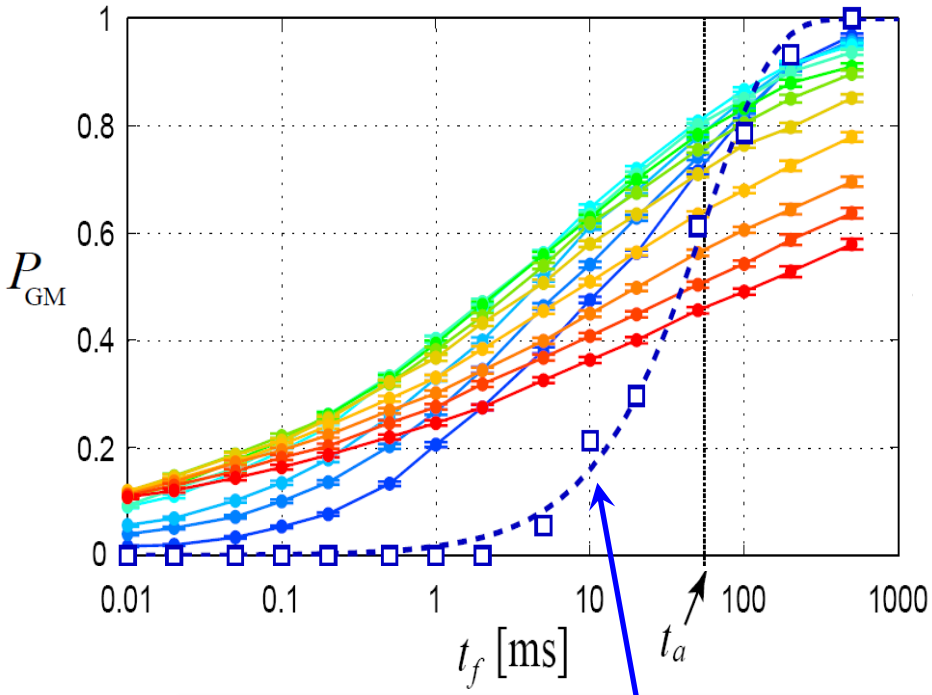
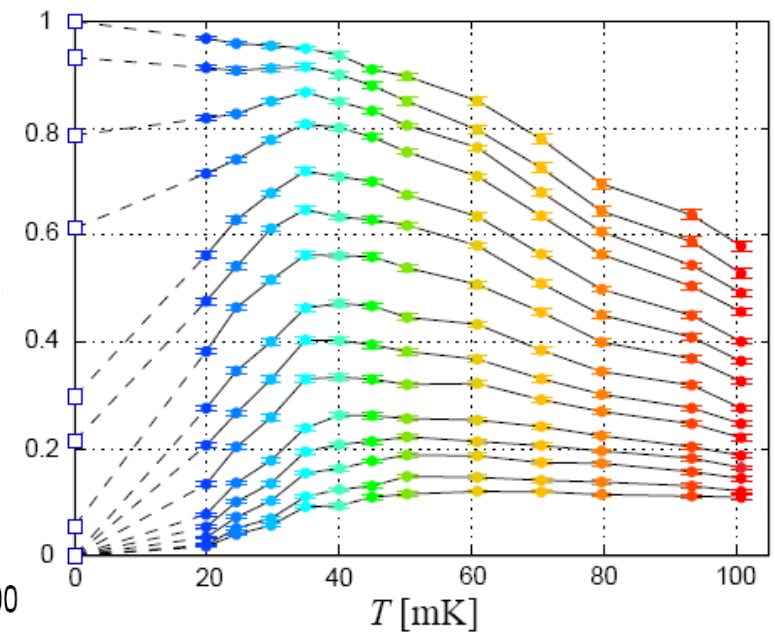
T=0 Behavior





t_f [ms]
500
200
100
50
20
10
5
2
1
0.5
0.2
0.1
0.05
0.02
0.01

Extrapolated T=0 points fit well with closed system (Landau-Zener) formula

All other curves fit very poorly

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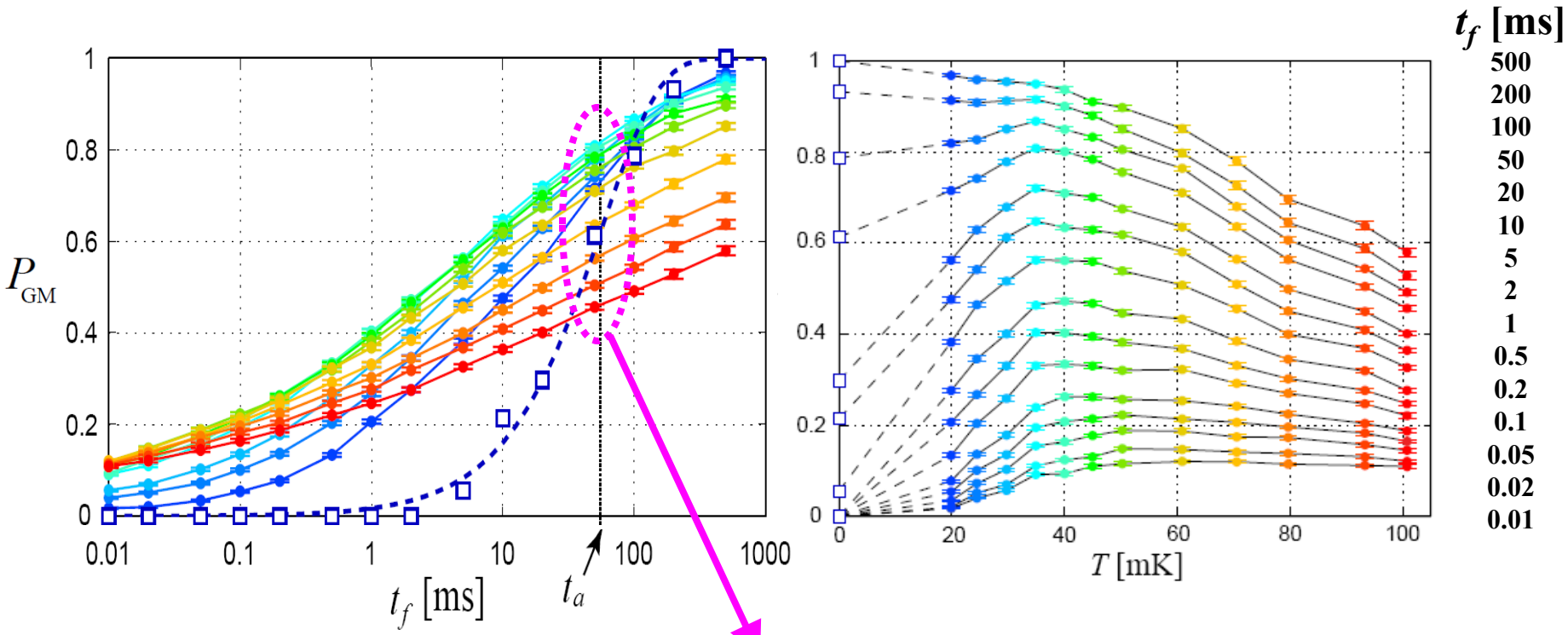
T=0 vs T>0 Behavior



Probabilities almost the same despite

$t_f >> t_{decoherence} \sim$ 5-10 ns

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Conclusions

1. AQC is has a natural robustness against decoherence

2. Environment sometimes may help the system to reach the ground state faster

3. Results are in agreement with quantum mechanics

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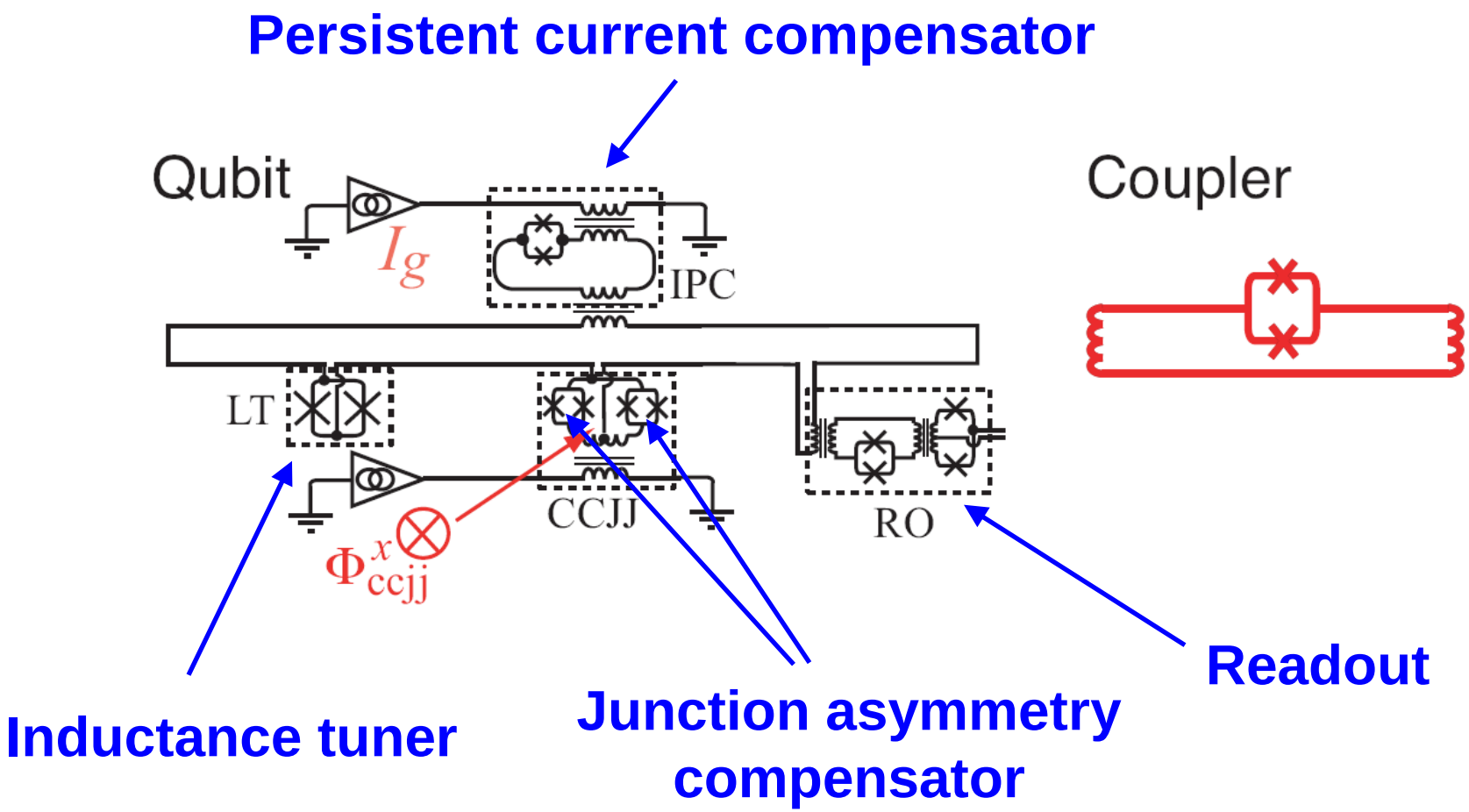
RF-SQUID Qubit

Harris et al., PRB 81, 134510 (2010)



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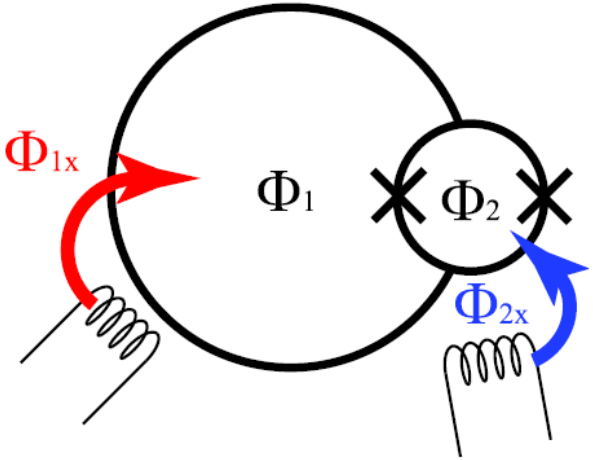
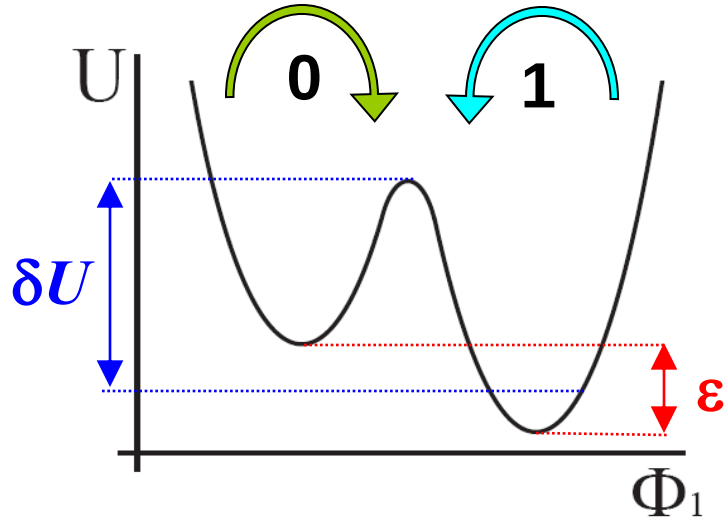
Simplified Qubit





$$H = -\frac{1}{2}\left[\varepsilon\sigma_z + \Delta(\delta U)\sigma_x\right]$$

Control knobs: **Φ_{1x} controls energy bias ε**

Φ_{2x} controls barrier height δU

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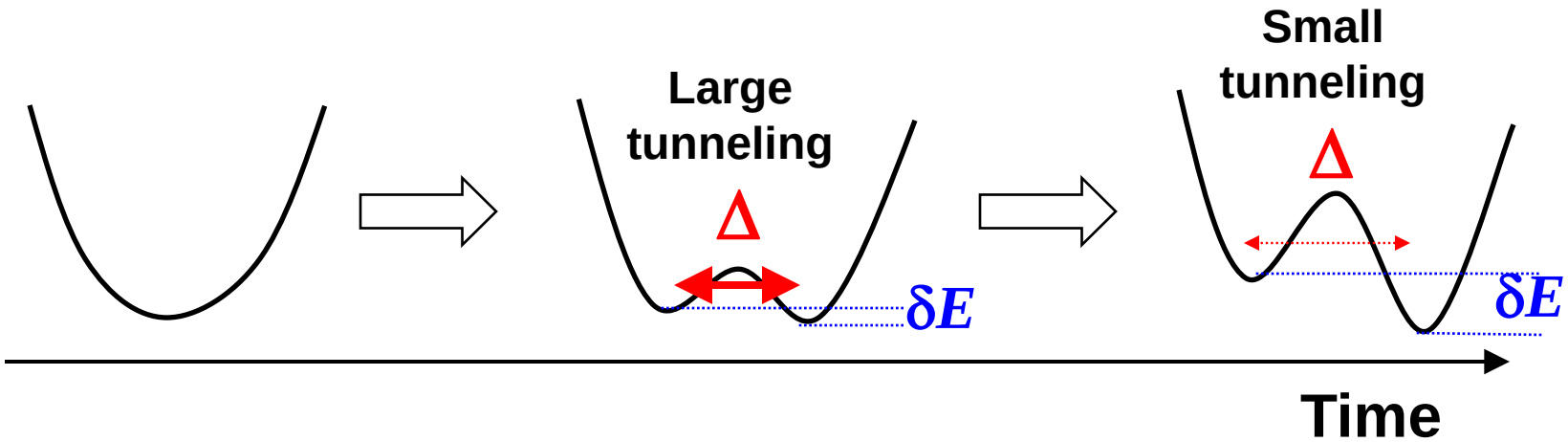
Adiabatic Evolution

Raising barrier height δU, decreases tunneling: $\Delta(t)$



Raising δU also changes energy scale: $\mathcal{E}(t)$

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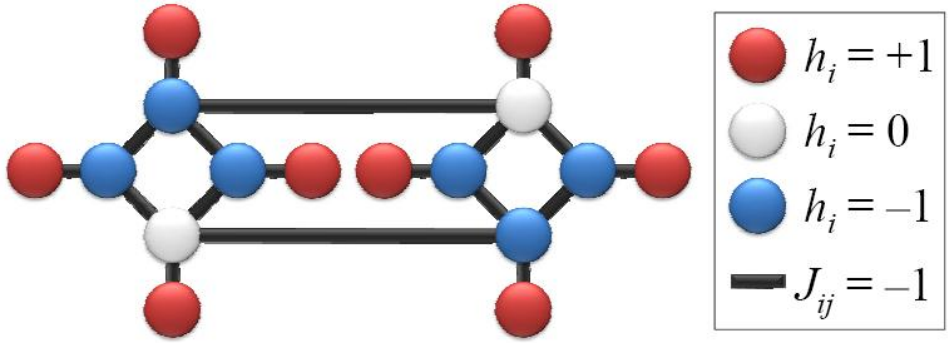
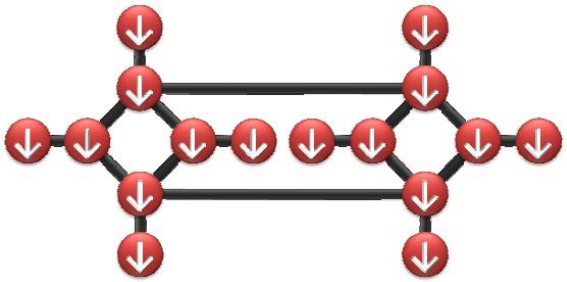
A 16-Qubit Small Gap Problem



$$H = -\frac{1}{2}\sum_{i=1}^{N}\Delta_i(s)\sigma_i^x + \frac{1}{2}\mathcal{E}(s)H_P$$



$|M\rangle$

Final ground state (global minimum of H_P)

There are 256 local minima of H_P

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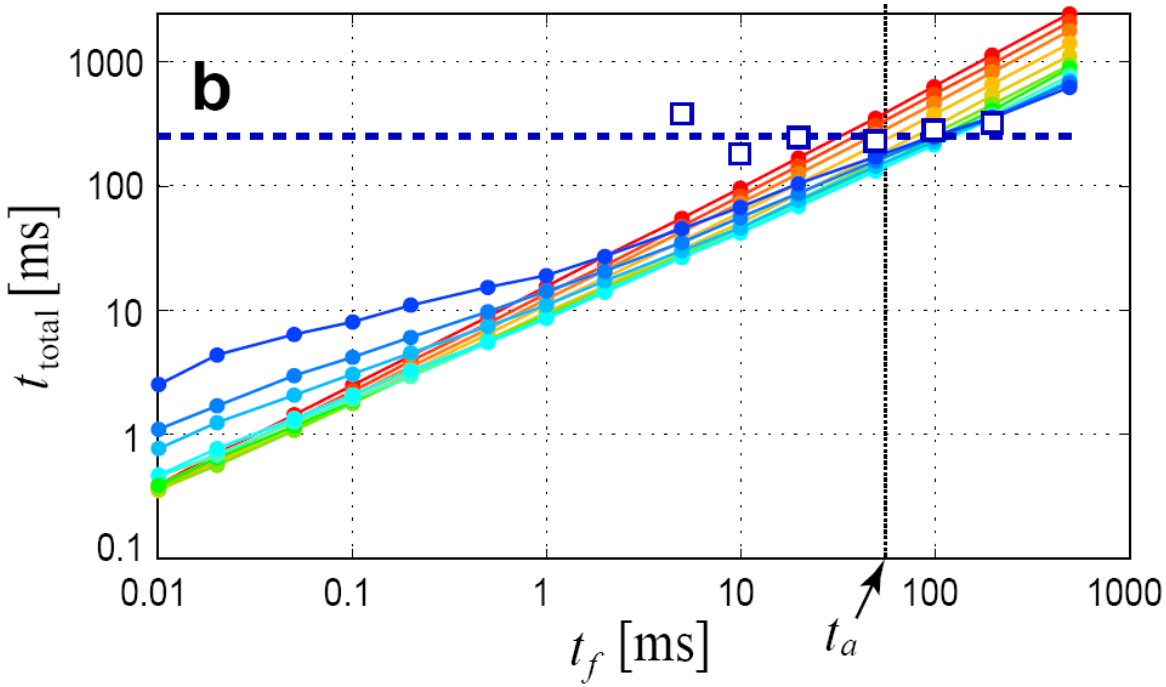
Speedup by Repetition

After k repetition: $P_{\text{total}} = 1 - (1 - P_{\text{GM}})^k, \qquad t_{\text{total}} = k t_f$

time to reach P_{total} = 0.99



Significant speedup can be achieved by multiple fast evolutions

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