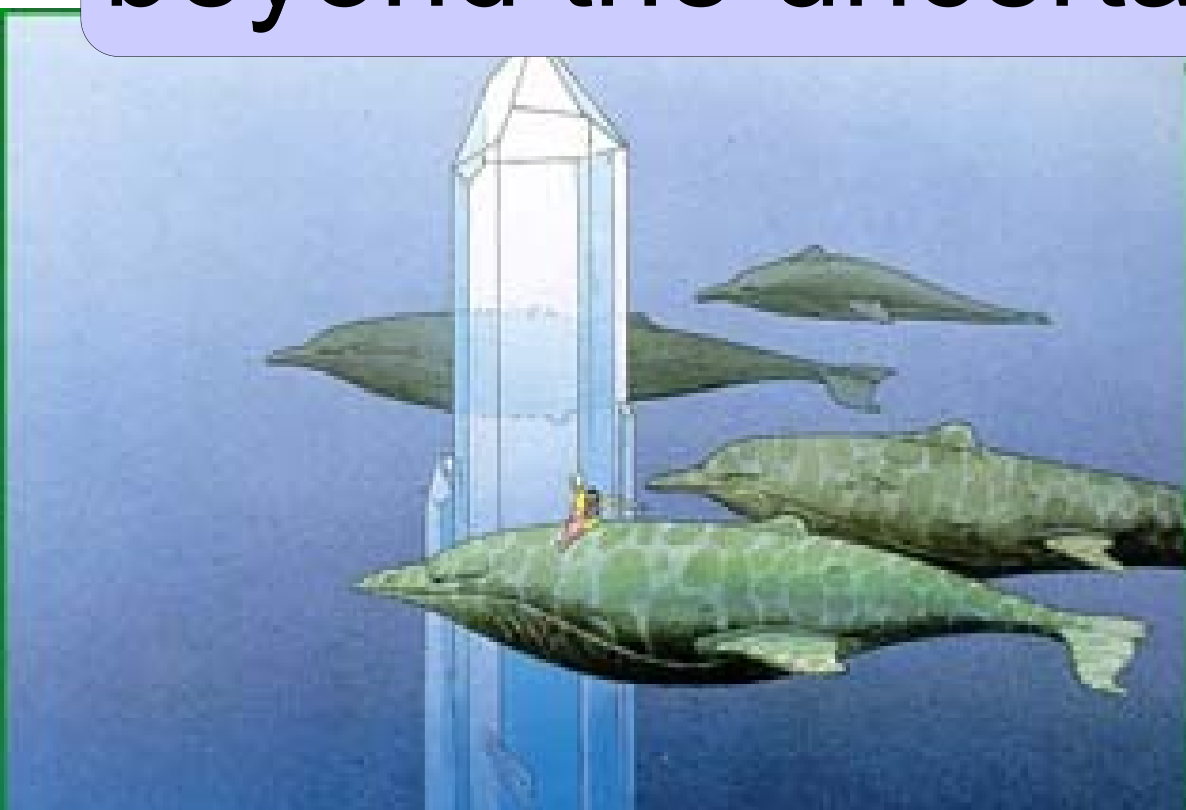
Quantum measurement bounds: beyond the uncertainty relations



maccone@unipv.it

Lorenzo Maccone
Dip. Fisica, INFN Sez. Pavia,
Universita' di Pavia

Vittorio Giovannetti
NEST-INFM & Scuola Normale Superiore,
Pisa, Italy

Seth Lloyd
Massachusetts Institute of Technology,
Cambridge (MA), USA.

QUit quantum information theory group
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What am I talking about?

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A new measurement-precision bound:

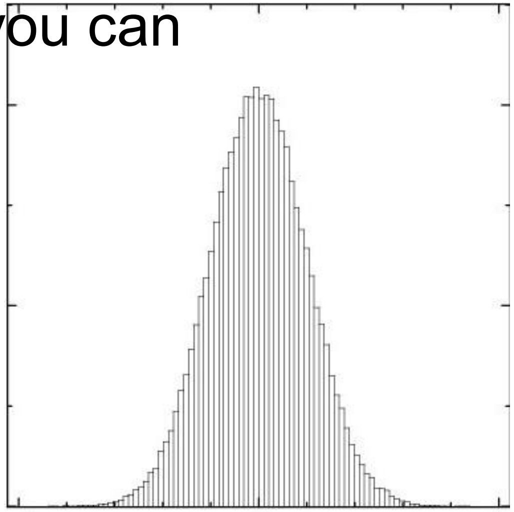
a generalization of Heisenberg-type uncertainty relations

Outline

- Heisenberg uncertainty relations and q. Cramer-Rao bounds
- Our bound: **a first moment generalization**
- Ideas behind the proof
- Prior information?

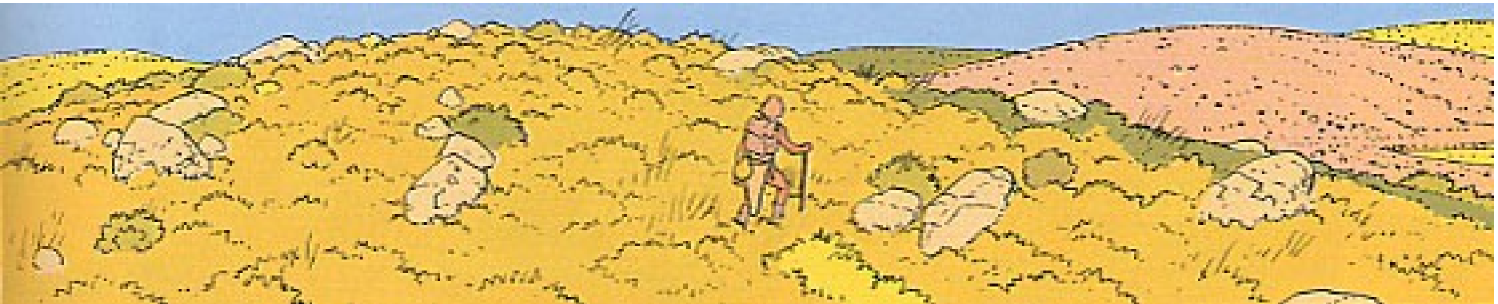
Abstract: In quantum mechanics, the Heisenberg uncertainty relations and the Cramer-Rao inequalities typically limit the precision in the estimation of a parameter through the *standard deviation* of a conjugate observable. Here we extend these relations by giving a bound to the precision of a parameter in terms of the *average value* of the conjugate observable. This has both foundational and practical consequences: in quantum optics it resolves a controversy on which is the ultimate precision in interferometry.

Heisenberg uncertainty relation



"If you have a probe system with spread Δp in momentum, you can discover its position with uncertainty Δx"

$$\Delta X \geqslant \frac{\hbar}{2\Delta p} \equiv \frac{1}{\Delta H}$$



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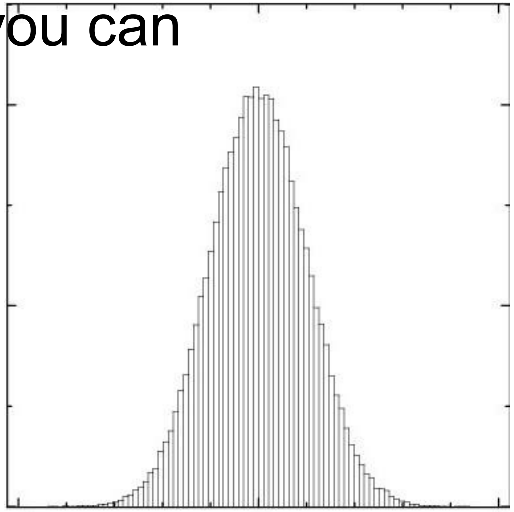
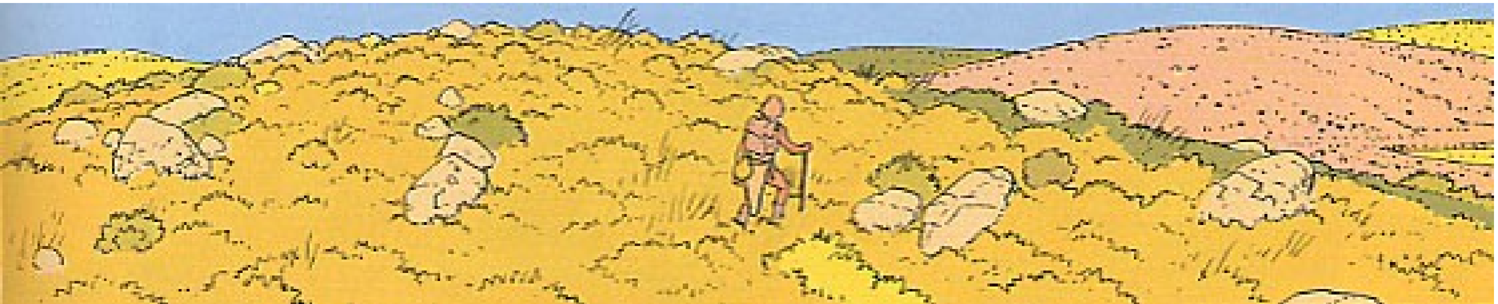


$$\Delta X \geqslant \frac{\hbar}{2\Delta p} \equiv \frac{1}{\Delta H}$$

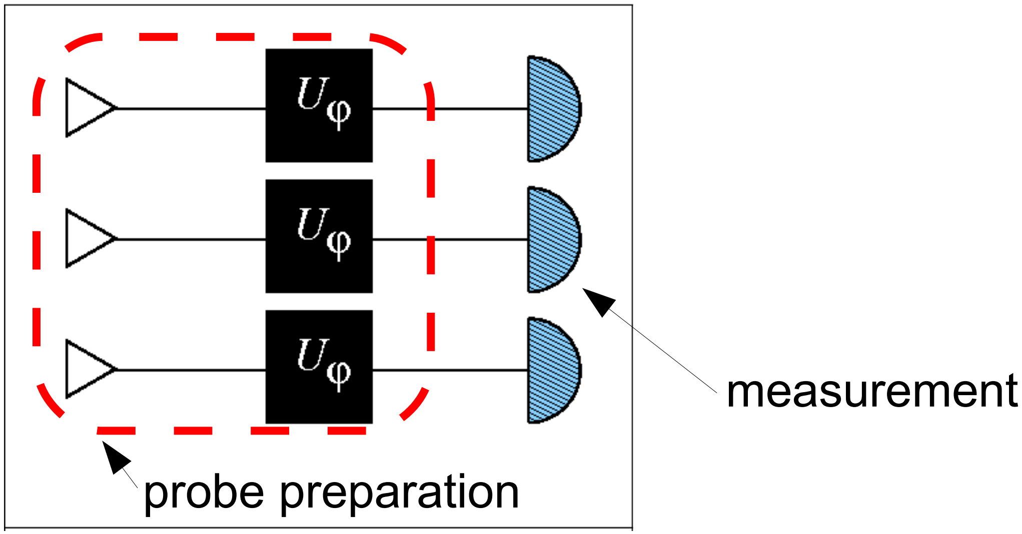
H is the generator of translations of *X*:

$$\rho_x = e^{-ixH} \rho_0 e^{ixH}$$



Heisenberg uncertainty

To increase precision, prepare and repeat the measurement ν times,



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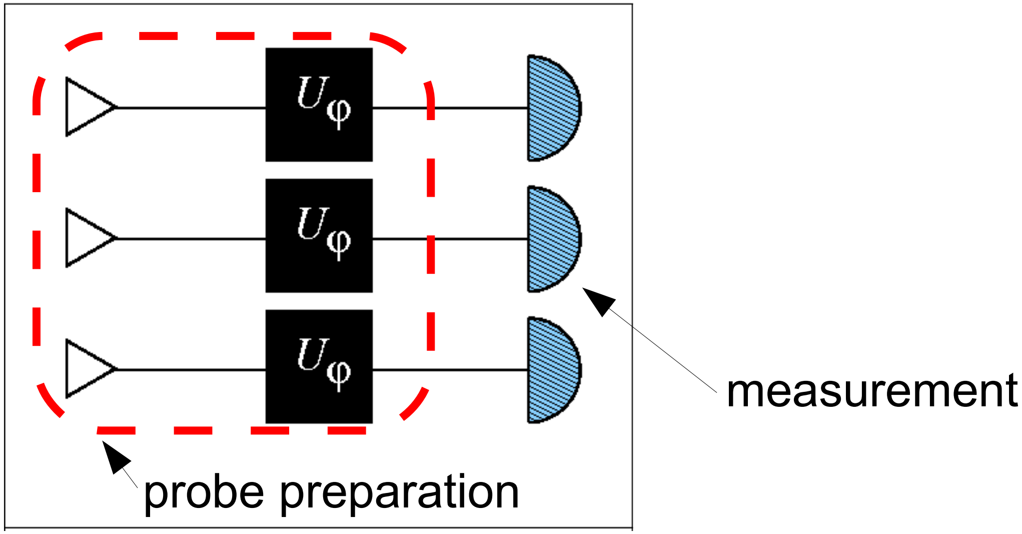

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(central limit theorem)

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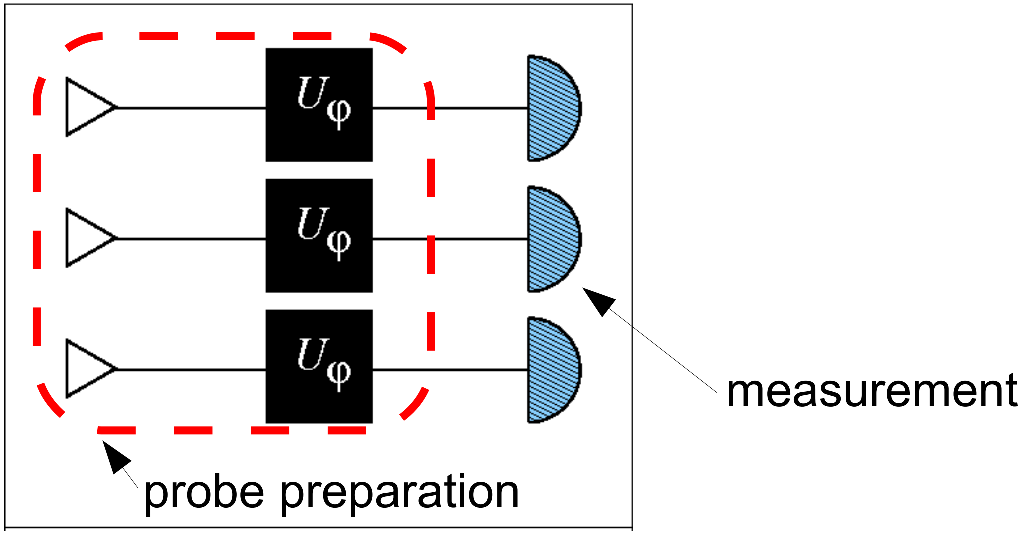

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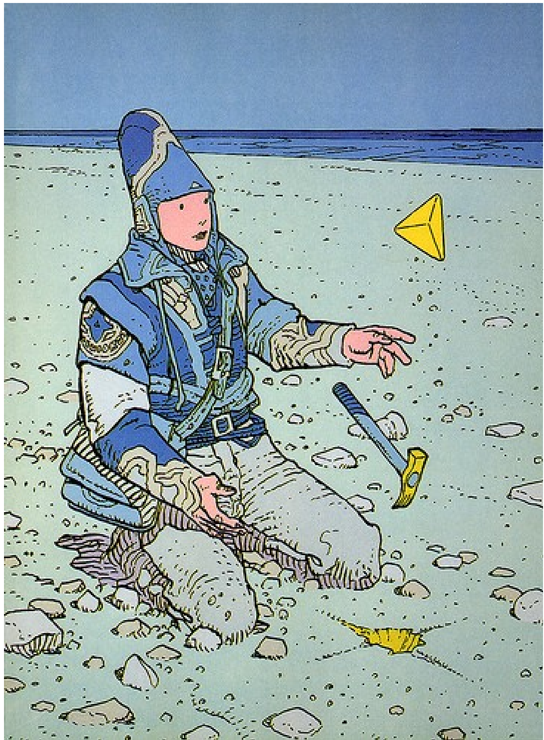
Remember, we're looking for the **AVERAGE** position (not *the* position)

Cramer-Rao bound

An **achievable** lower bound on the precision for the measurement of a **parameter**



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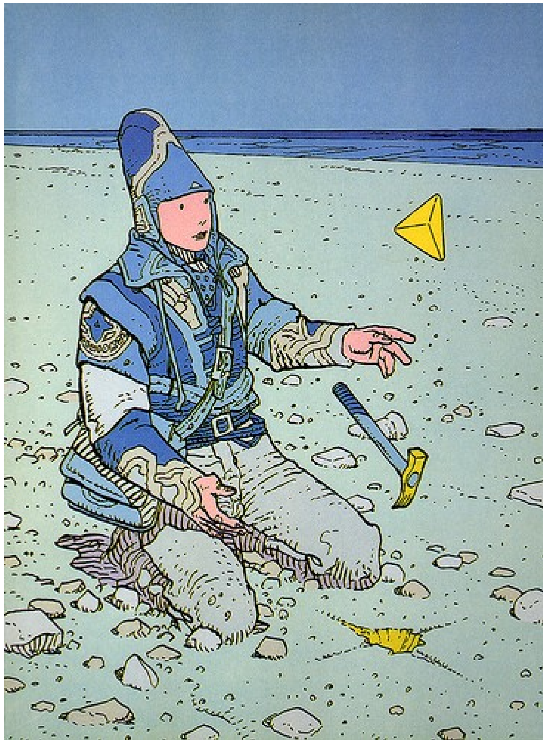

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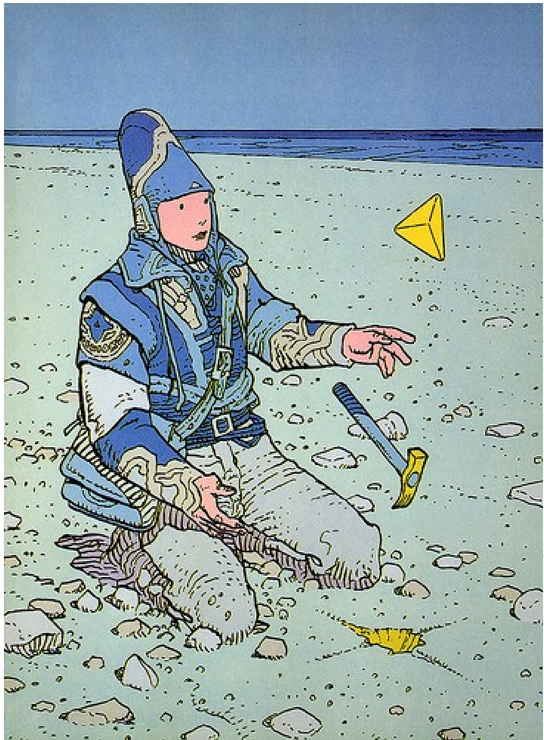

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but now *x* is a **parameter** (not an observable)

and again H is the generator of translations of *x*:

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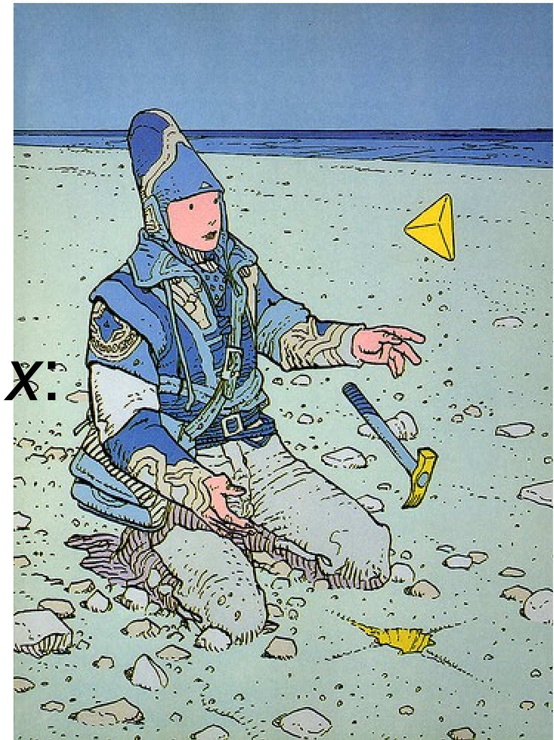

New bound

Heisenberg / Cramer-Rao

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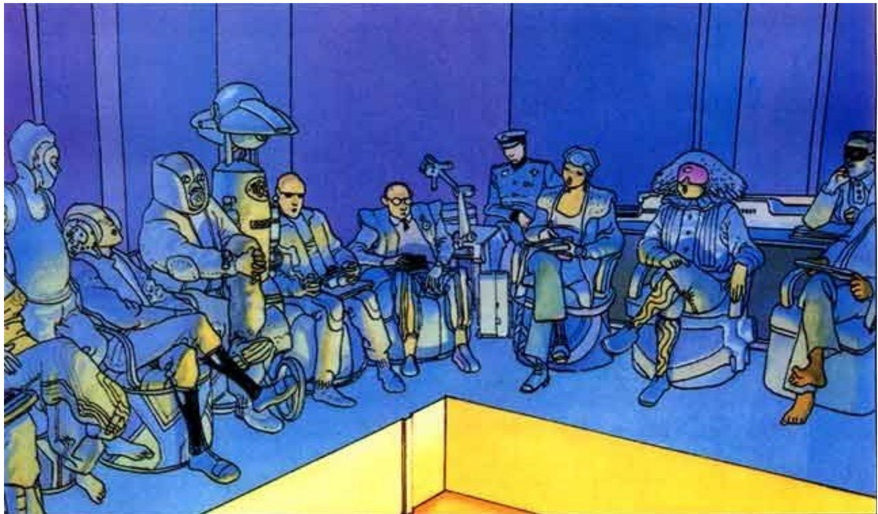
a first moment generalization of

Heisenberg uncert. / Cramer-Rao bound



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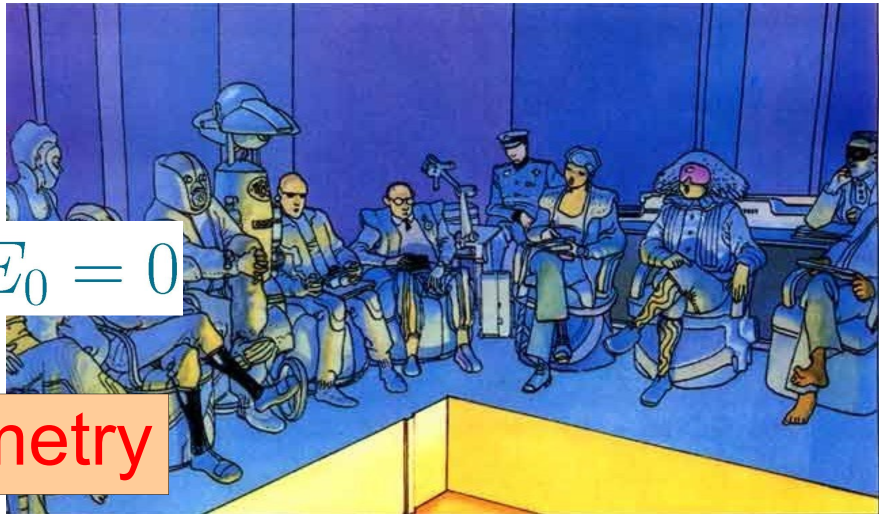
a first moment generalization of Heisenberg uncert. / Cramer-Rao bound

For interferometry:

$$H = a^\dagger a \ , \ \ x = \phi \ , \ \ \nu = 1$$

$$\boxed{\Delta \phi \geqslant \frac{\kappa}{N}}$$

$$N = \langle a^\dagger a \rangle \ , \ \ E_0 = 0$$



The Heisenberg bound for interferometry

People are saying the Heisenberg bound **CAN** be beaten...

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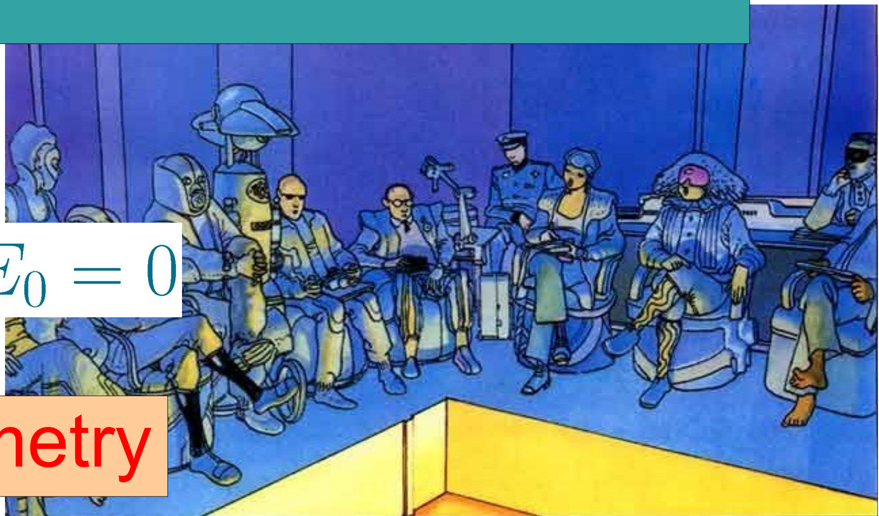

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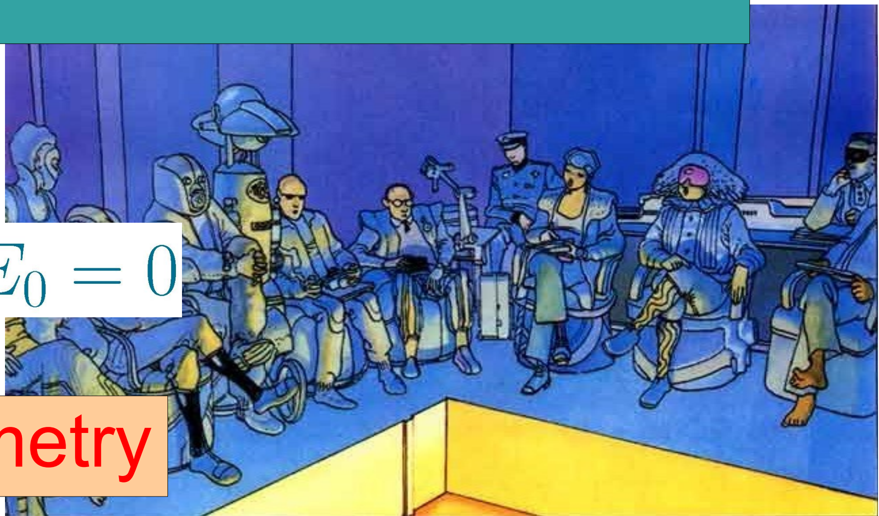

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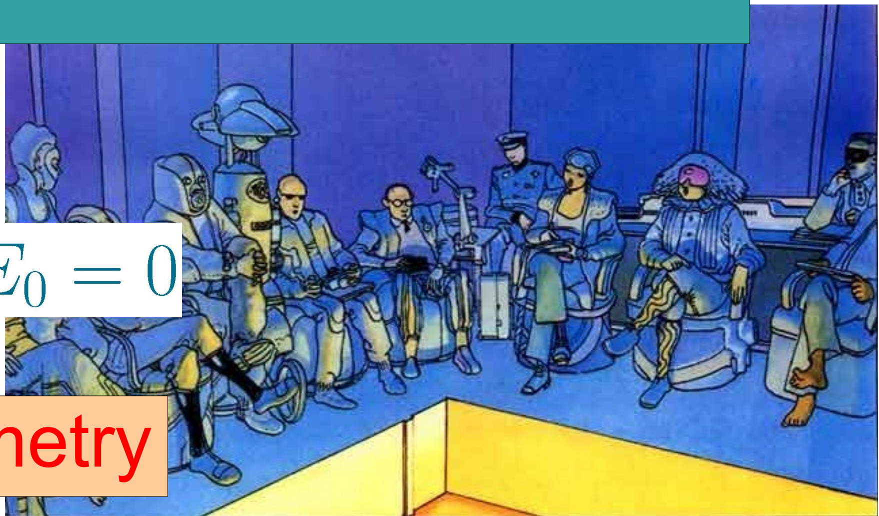

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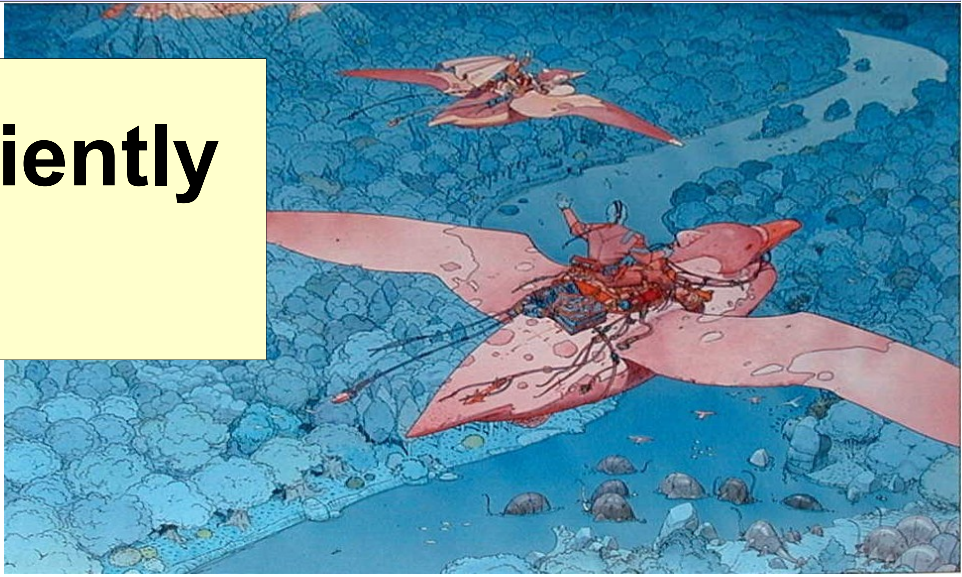
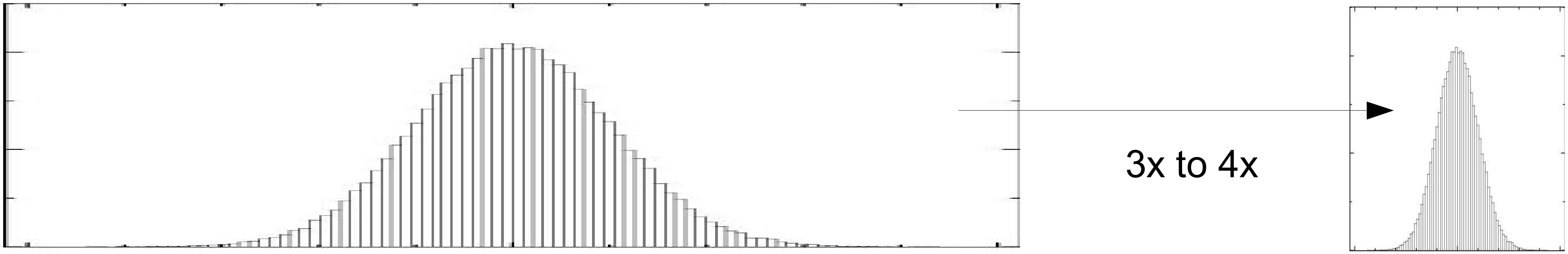
Hypothesis behind our bound



Our bound works for sufficiently good estimation strategies

i.e. that give much better
precision than the prior information:

(I'll get back to this later)



(so our bound CAN be beaten if the estimation strategy is BAD:
if you have LOTS OF PRIOR INFORMATION)

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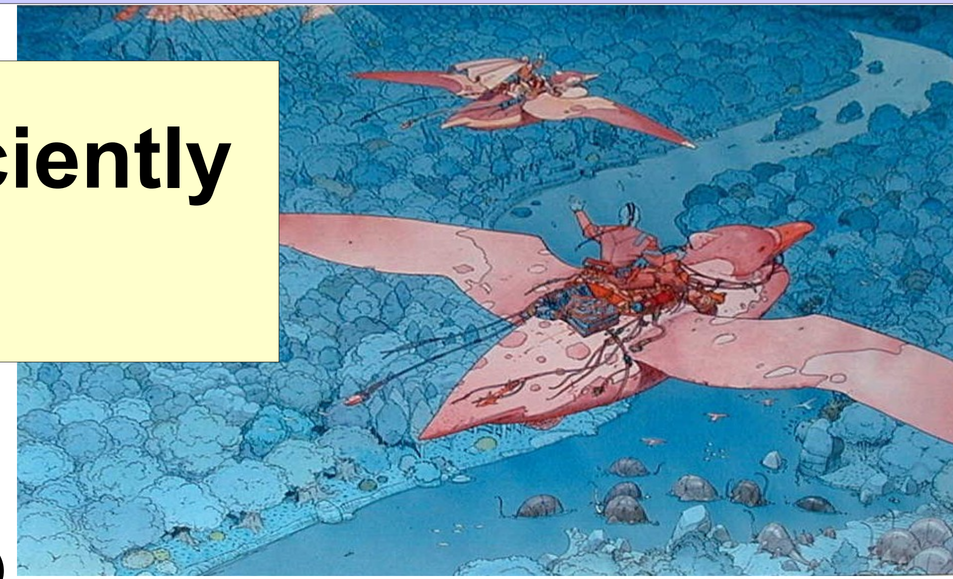

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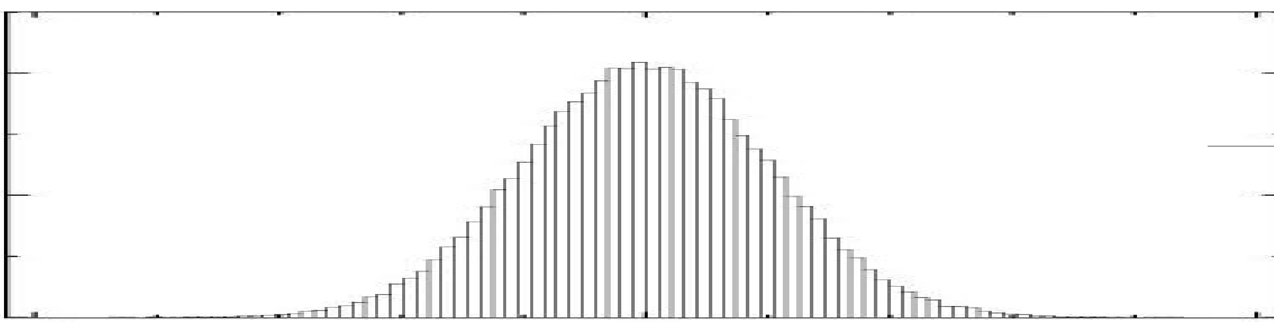

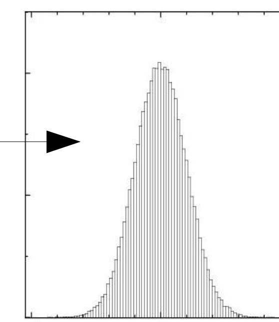

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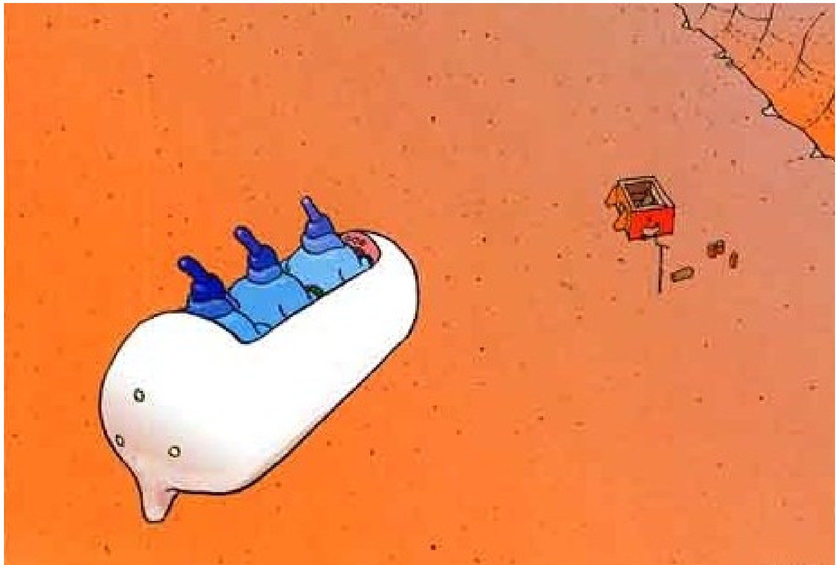
UNINTERESTING strategies

Parameter-dependent error

..but ΔX may depend on the (true value of the) parameter x

(so the error can be zero for a specific value of x; e.g. a broken clock!)



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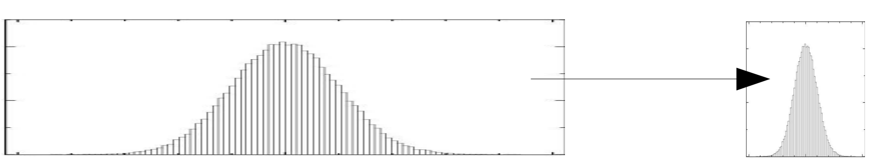
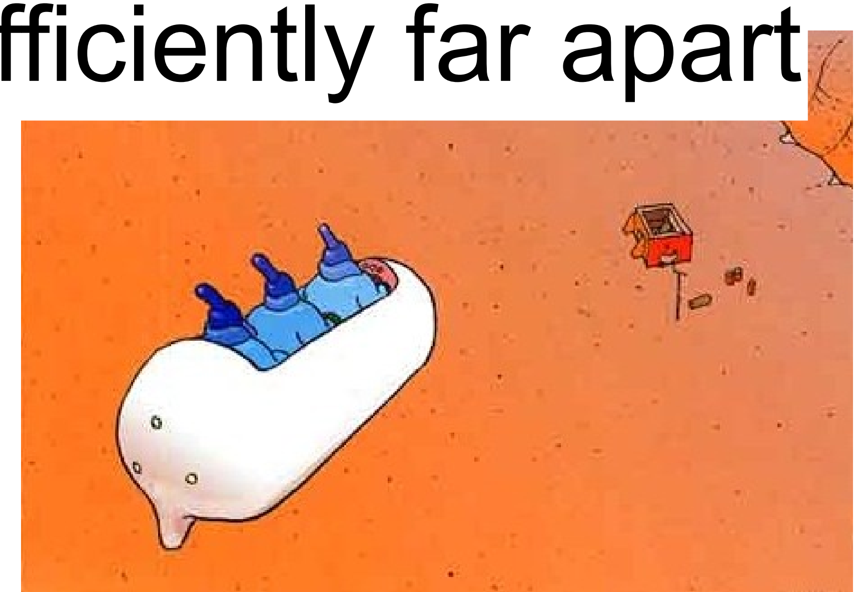
$\Rightarrow$ our bound holds for the **average**

$$\boxed{\frac{\Delta X(x)+\Delta X(x')}{2} \geqslant \frac{\kappa}{\sqrt{\nu}\mathcal{H}}}$$

IF we can find x and x' sufficiently far apart

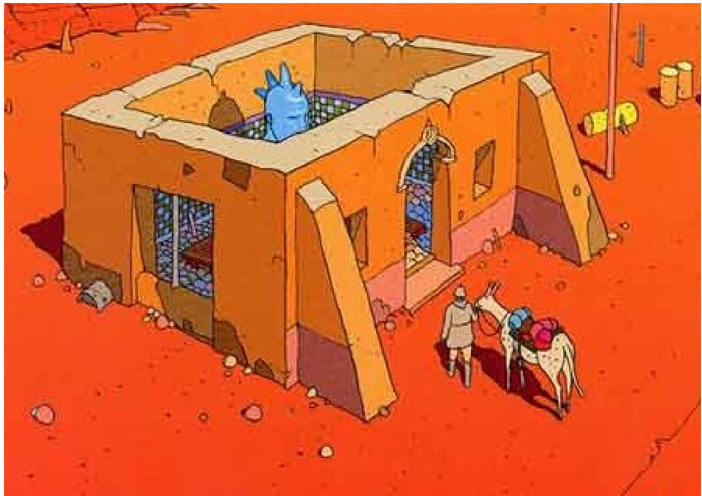
we can always do that if the estimation strategy is good enough, i.e. it yields much better precision than the prior information:





Generality of our bound

The bound holds generally:

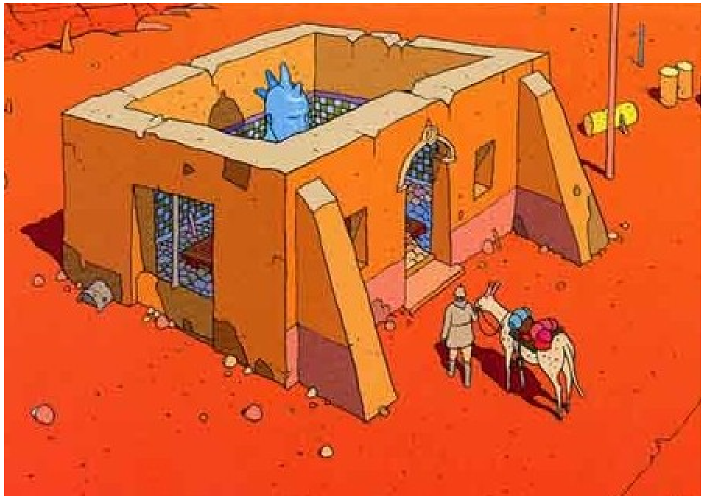


The bound holds generally:

- Unbiased or biased measurement procedures

- Local or global estimation strategies

- Pure or mixed state of the probes.



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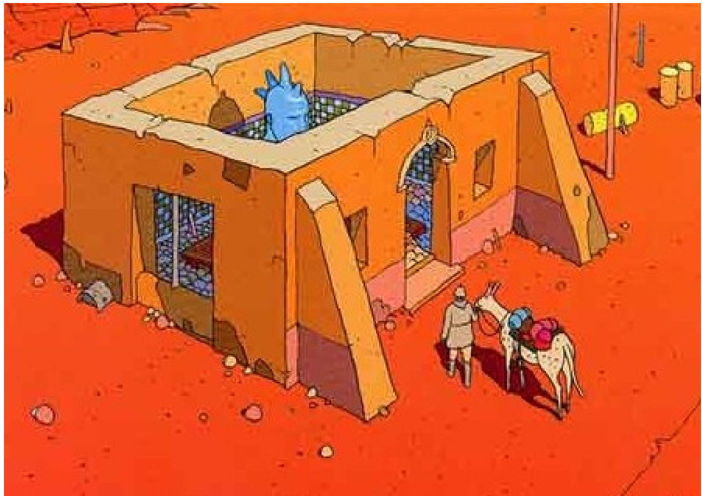

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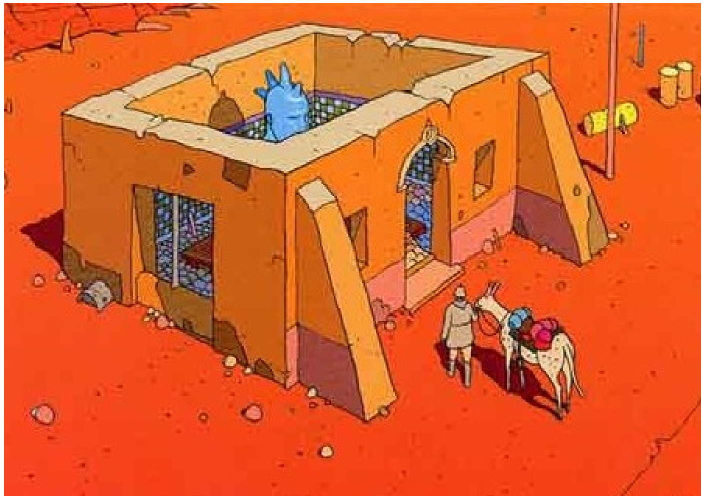

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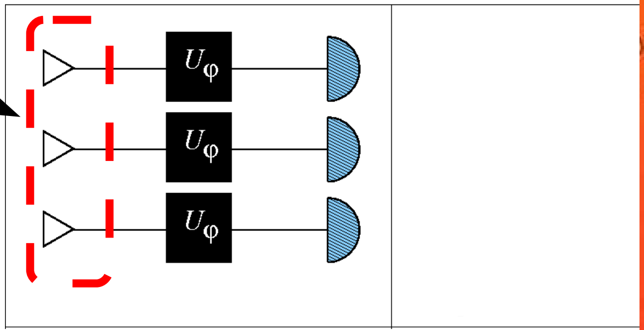

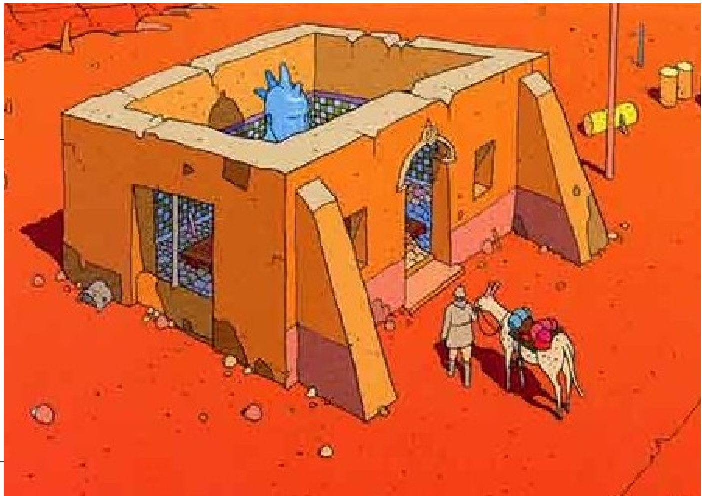

PROOF: main idea

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$$p(y) = \left|\langle y|[|\psi(x)\rangle]^{\otimes\nu}\right|^2$$

prob. of obtaining *y* when the true value of the ν probes was *x*

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Choose x, x' such that $x' - x = 2\lambda\Delta X$ $\lambda \gtrsim 1$

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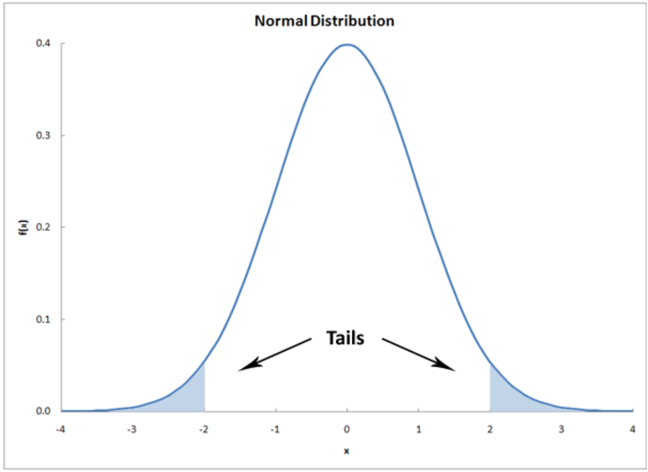

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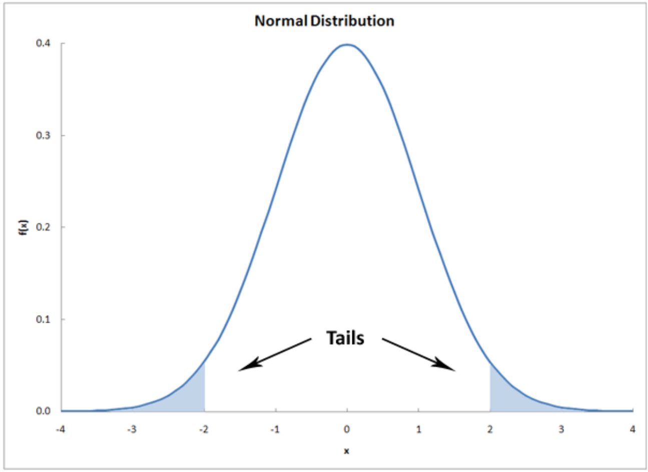

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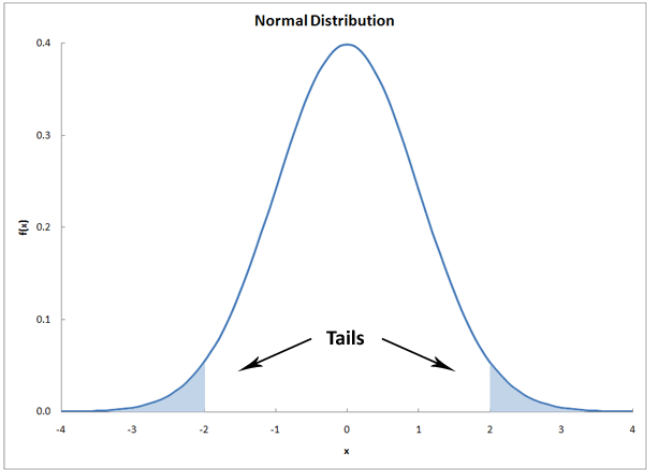

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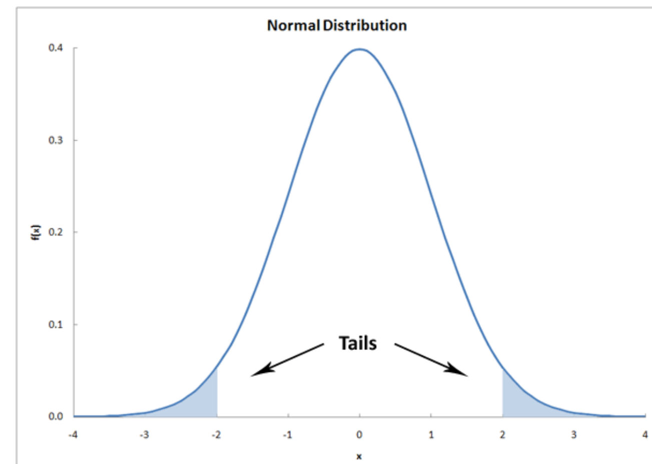

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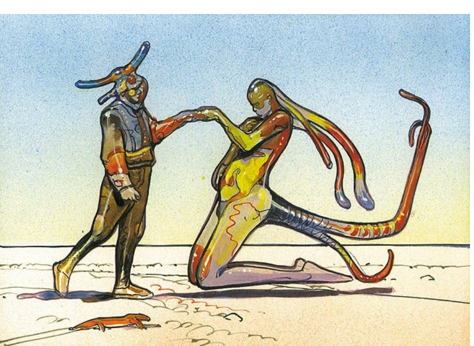
$\Rightarrow$ the overlap cannot be too large!



$$F = |\langle\psi(x)|\psi(x')\rangle|^{2\nu} \leqslant \frac{4}{\lambda^2}$$

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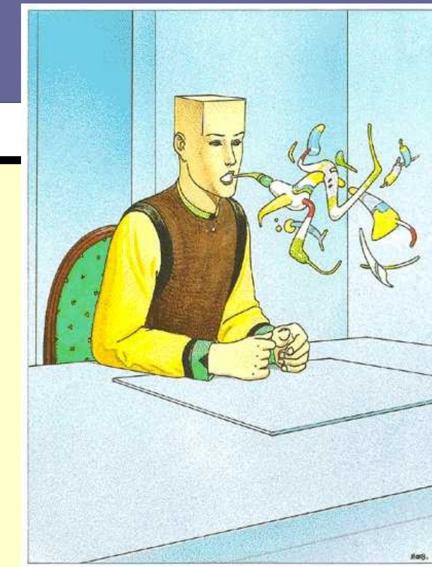

LOST?

Cl

Tch

$\Rightarrow$

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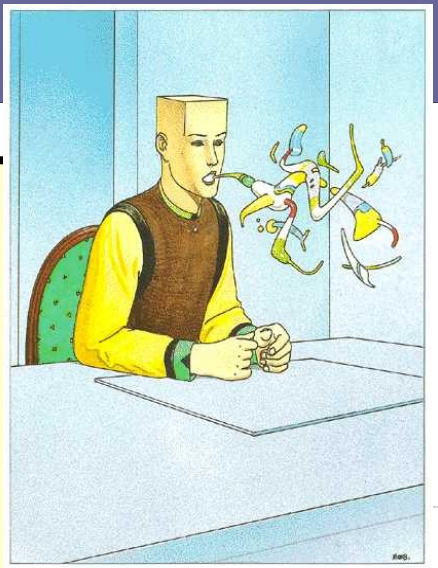

Ch

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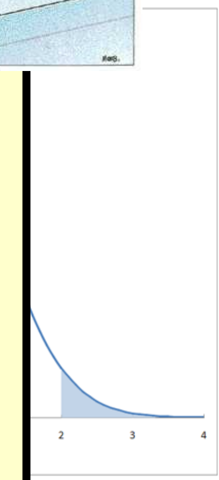
connect the error ΔX to the fidelity F :

$$x' - x = 2\lambda\Delta X \Rightarrow F \leqslant \frac{4}{\lambda^2}$$



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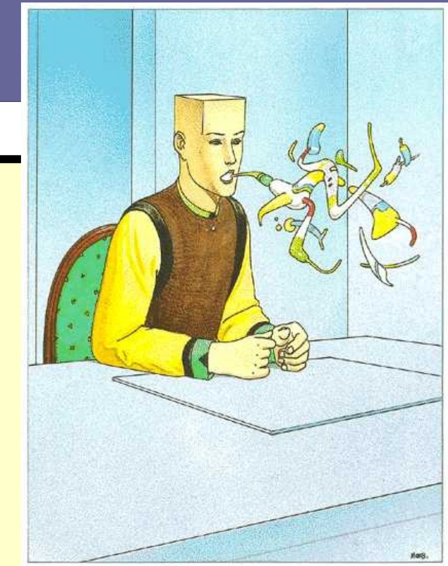

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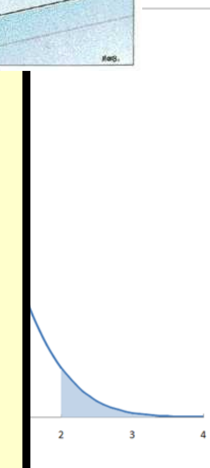
now, we have to connect the fidelity with the expectation value $\mathcal{H}$ of the translation generator



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$$F \equiv |\langle\psi(x)|\psi(x')\rangle|^2 = \epsilon \text{ ?}$$

(i.e. how "fast" can a state evolve?)

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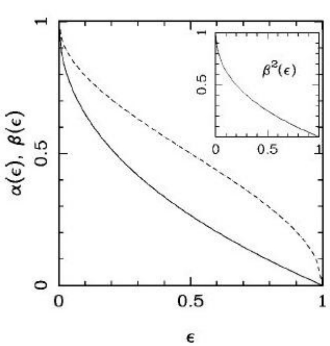

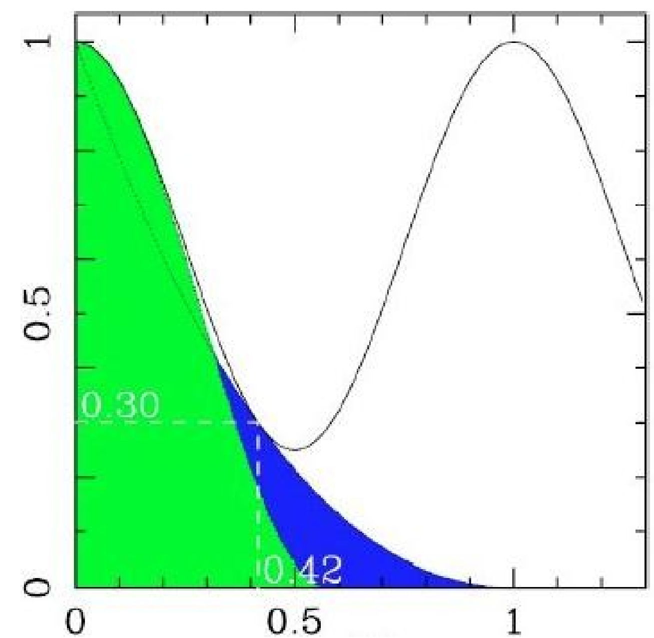

Use the q. speed limit

Consider an "evolution" $|\psi(x')\rangle = e^{-i(x'-x)H}|\psi(x)\rangle$, what is the smallest "time" x' at which the fidelity

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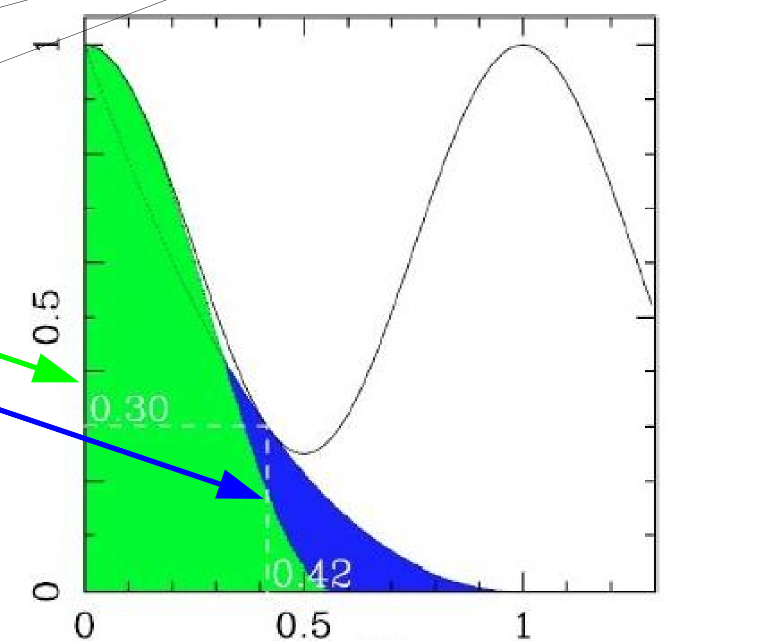

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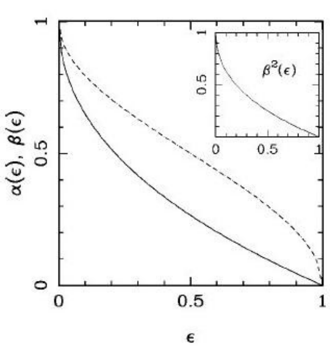

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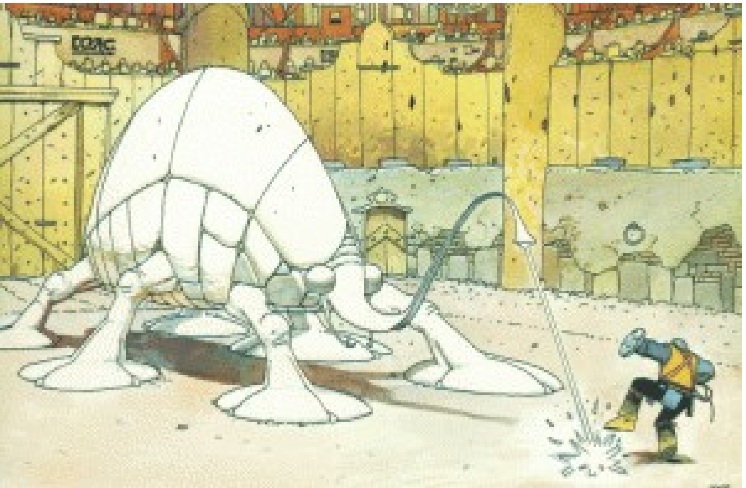

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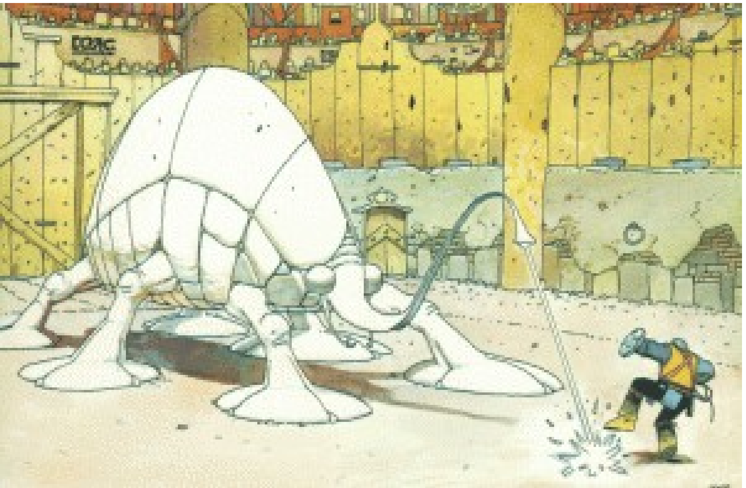

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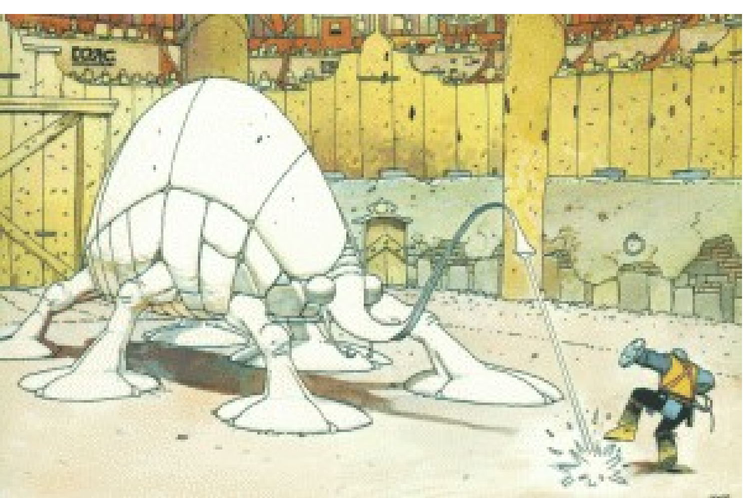
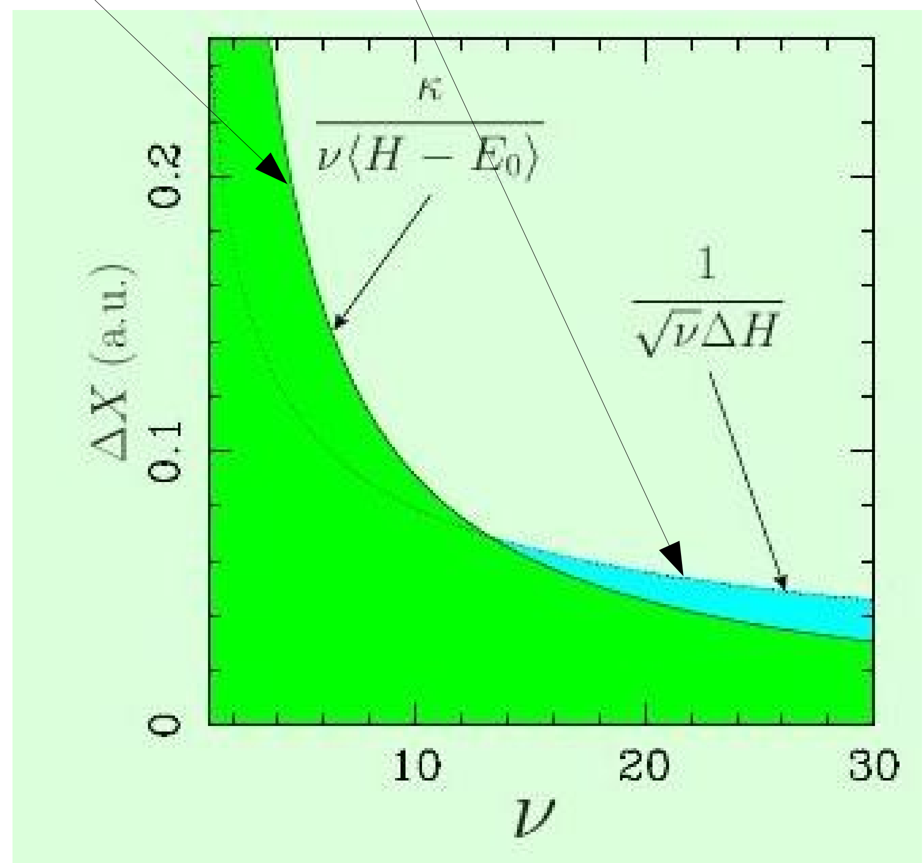
This is important → Cramer-Rao wins where it is physically significant, for $\nu \to \infty$





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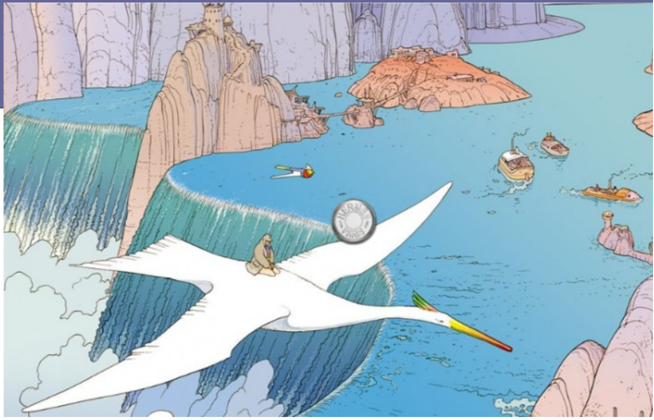

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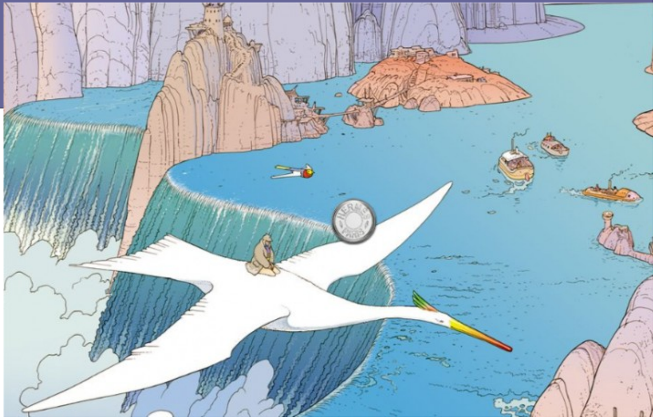

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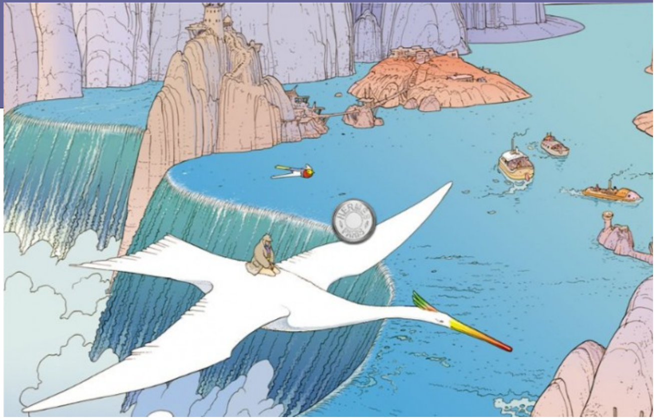

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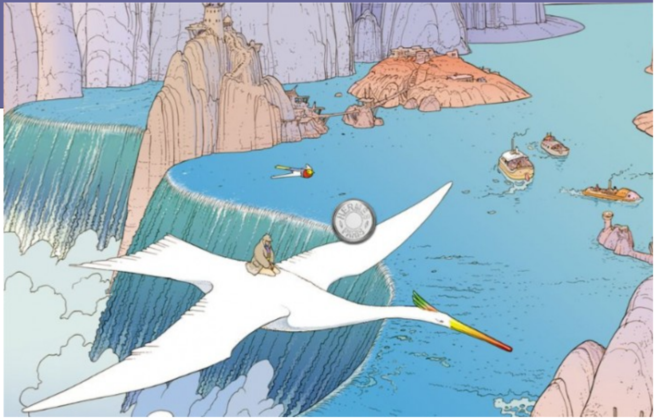

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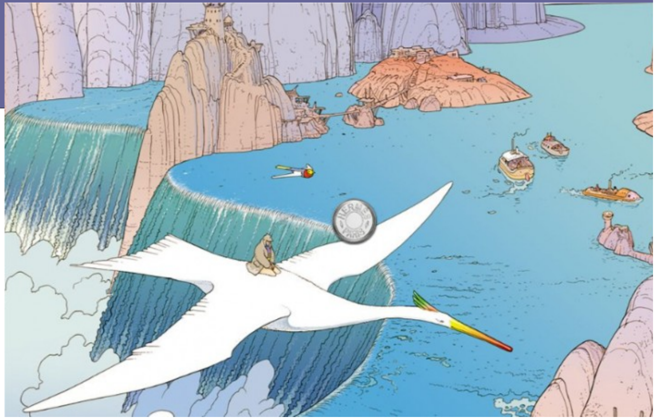

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(and also a Heisenberg-uncertainty bound)

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Can we always choose the appropriate λ?

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e.g., suppose we KNOW the value of x : there cannot be any bound on the precision of the determination!!! We know it already!

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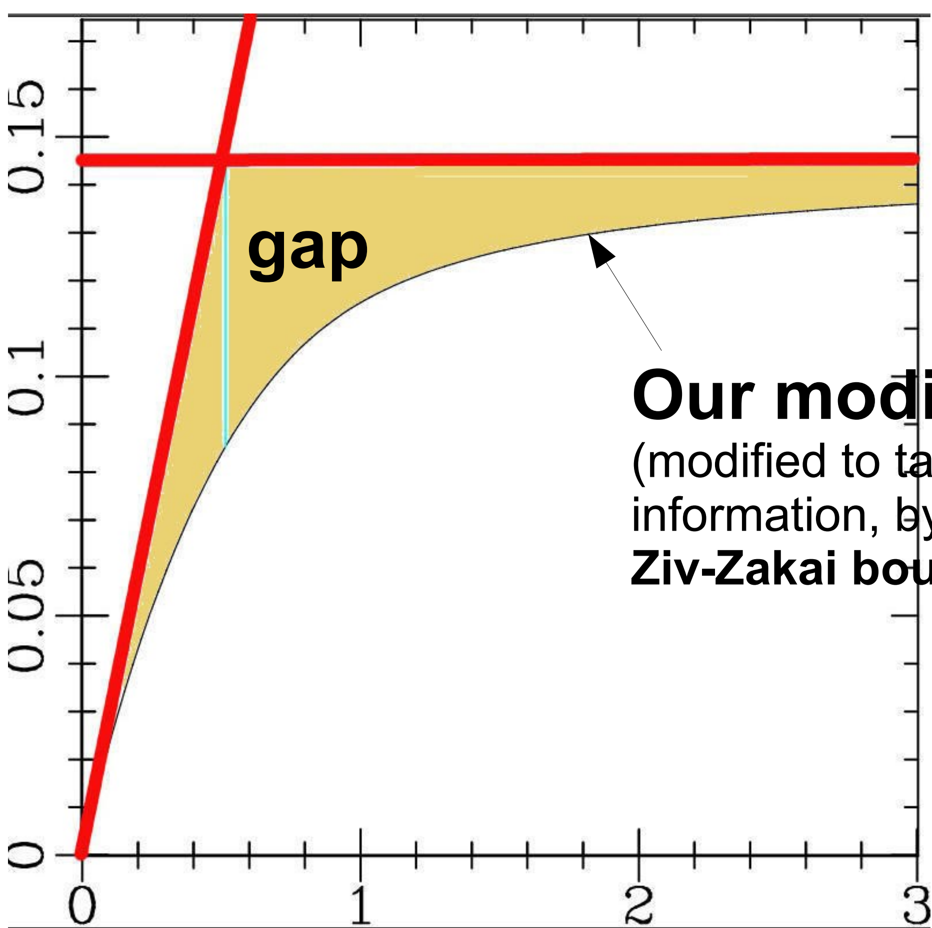

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(modified to take into account the prior information, by joining our bound to the quantum **Ziv-Zakai bound** [Tsang, arXiv:1111.3568])

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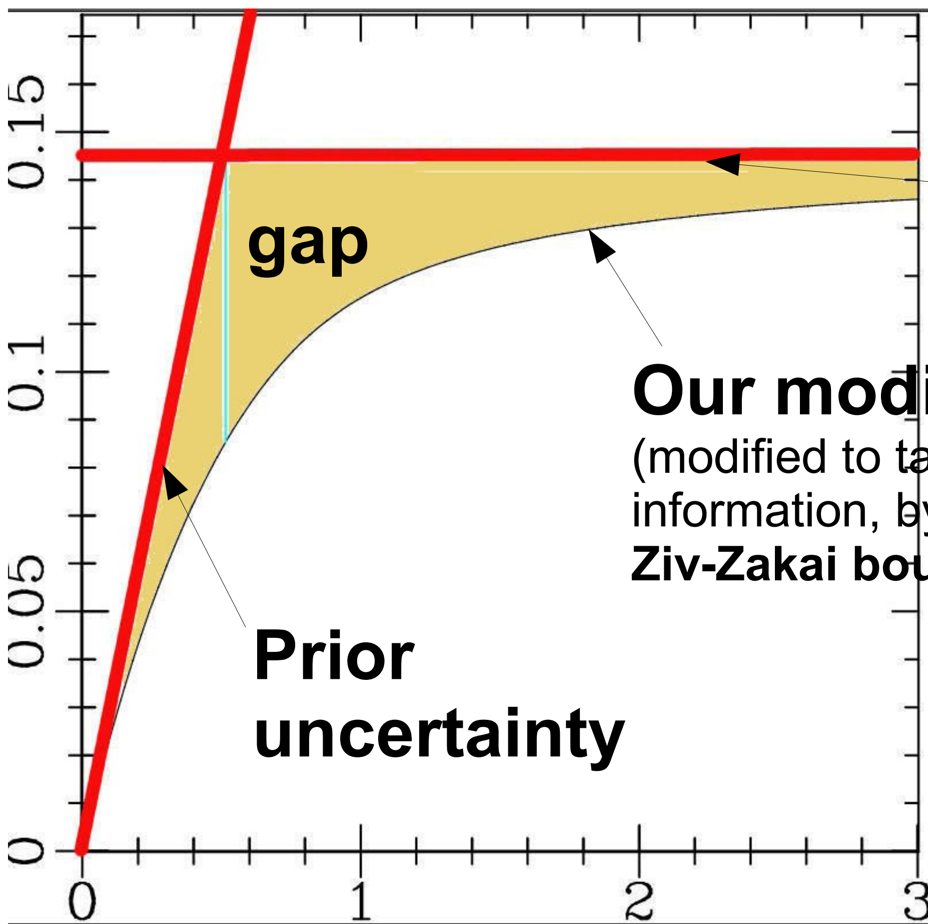

Heisenberg scaling

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$\Rightarrow$ **sub-Heisenberg strategies are useless**

Incidentally...

Ziv-Zakai bound

+

Heisenberg uncertainty/Cramer-Rao part of our bound

Prior information

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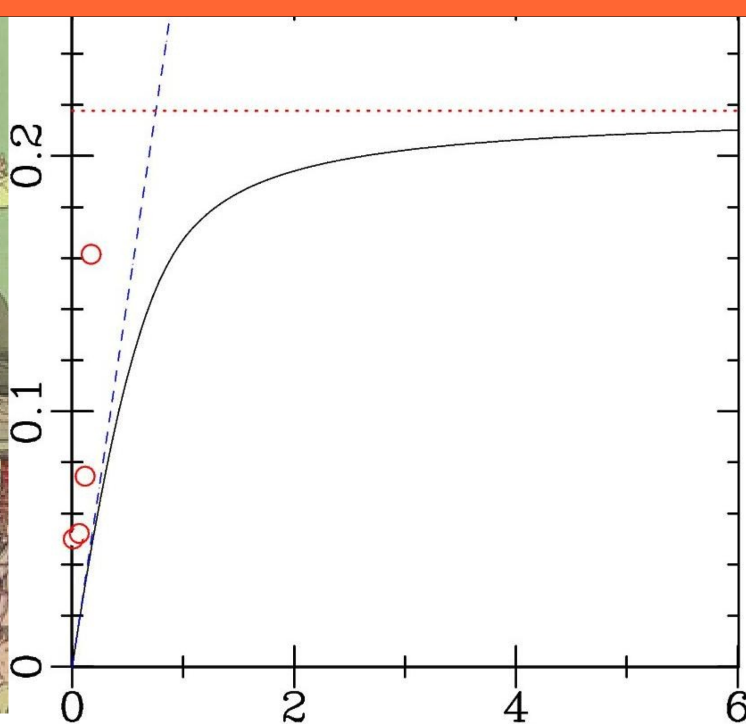
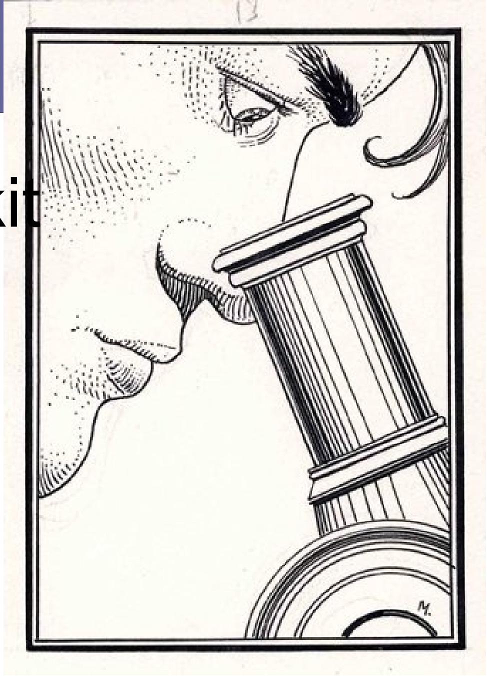
= an uncertainty relation weighted by an arbitrary prior distribution!



(Tsang already gave the case of uniform distribution)

Quantum Metrology

Our bound is part of the **quantum metrology** toolkit



Curious? → Recent review:

"Advances in Quantum Metrology", Nature Photonics **5**, 222 (2011).

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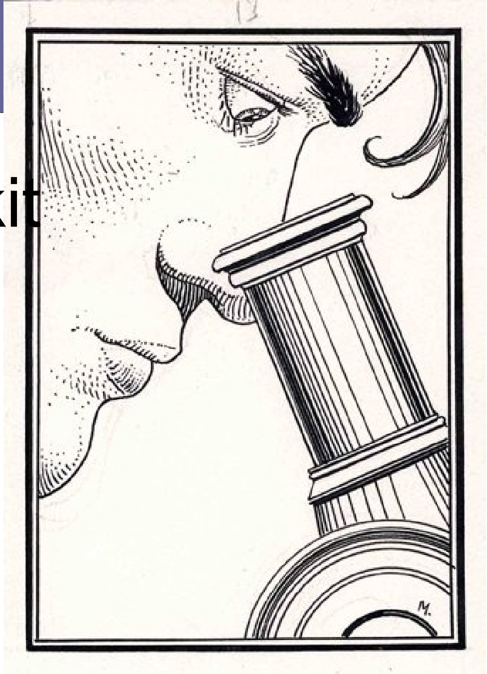

Quantum-enhanced measurement procedures (entanglement/squeezing)

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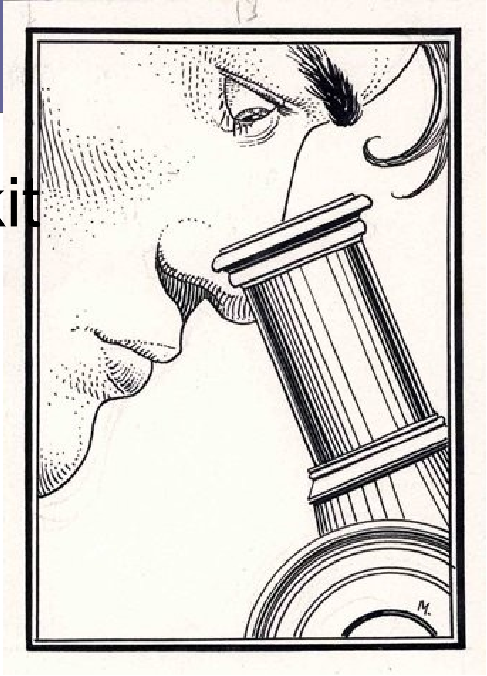

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Determining the ultimate bounds in measurements / measurement problem

quantum foundations

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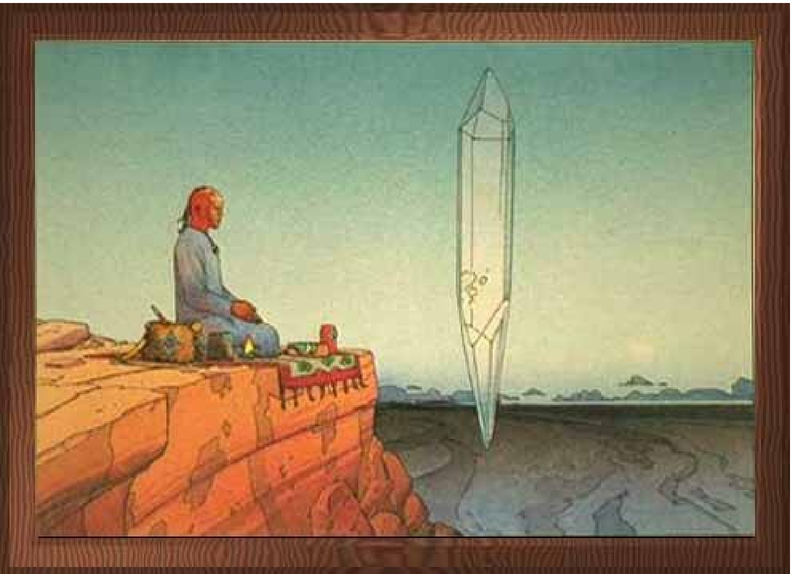
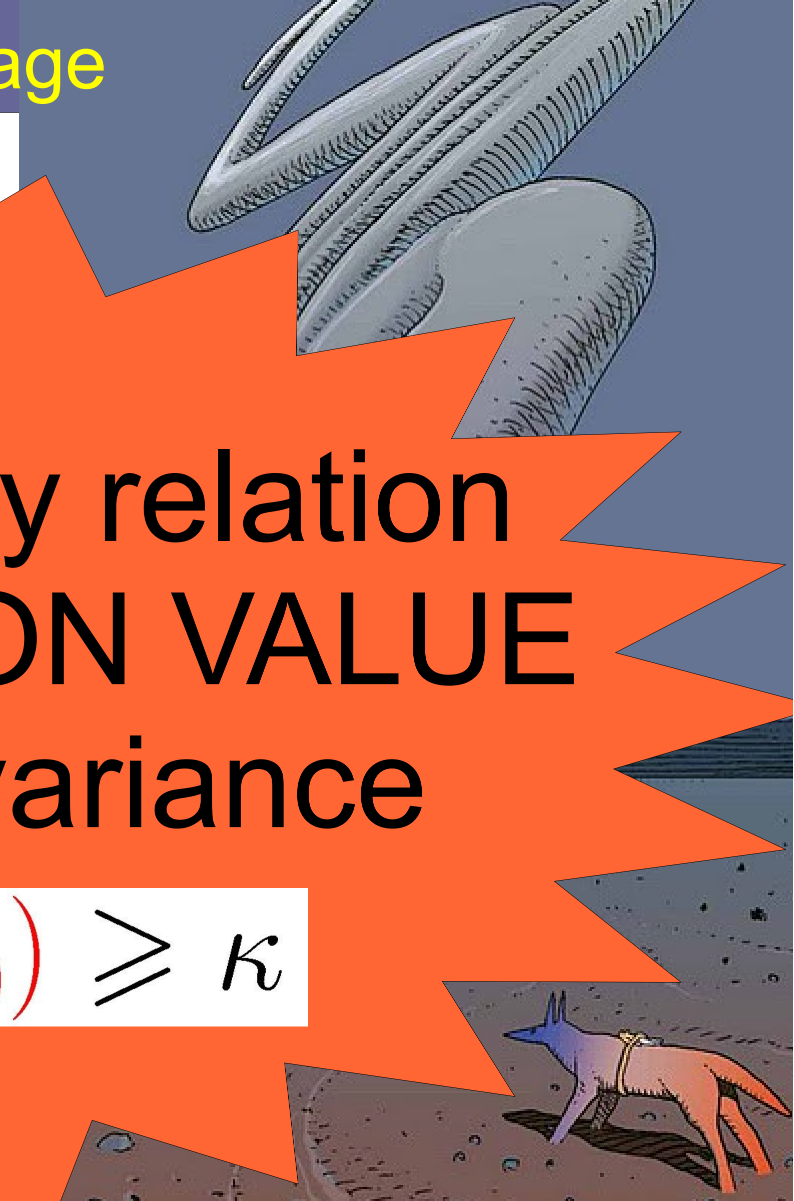
What did I say?!?

- Heisenberg uncertainty relations and q. Cramer-Rao bounds
- Our bound: **a first moment generalization**
- Ideas behind the proof
- Prior information $\Rightarrow$ sub-Heisenberg strategies exist, but are useless.



Take home message



A new uncertainty relation with EXPECTATION VALUE instead of the variance

$$\Delta X\left(\langle H\rangle - E_0\right) \geqslant \kappa$$

Bound: arXiv:1109.5661
Prior info: arXiv:1201.1878